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Quiz 13
You must show all work to receive full credit!!

Problem 1. (5 points) Evaluate $\int_C \mathbf{F}d\mathbf{r}$, where $\mathbf{F} = \langle \sin(y), x\cos(y) + \cos(z), -y\sin(z)\rangle$ and C is parameterized by $\mathbf{r}(t) = \langle \pi\sin(t), t, 2t\rangle$ for $0 \le t \le \pi/2$.

$\frac{\partial F_1}{\partial y} = \cos y = \frac{\partial F_2}{\partial x}$, $\frac{\partial F_2}{\partial z} = -\sin z = \frac{\partial F_3}{\partial y}$, $\frac{\partial F_3}{\partial x} = 0 = \frac{\partial F_1}{\partial z}$, so $\hat{F}$ is conservative.

$\int \sin y \, dx = x\sin y + f(y,z)$

$\int (x\cos y + \cos z)\, dy = x\sin y + y\cos z + g(x,z)$

$\int -y\sin z \, dz = y\cos z + h(x,y)$

$\Rightarrow f(x,y,z) = x\sin y + y\cos z$ is a potential function for $\hat{F}$.

Thus, by the Fundamental Thm we have

$$\int_C \hat{F}\, d\hat{r} = f(\hat{r}(\tfrac{\pi}{2})) - f(\hat{r}(0)) = f(\pi, \tfrac{\pi}{2}, \pi) - f(0,0,0)$$
$$= \pi + \tfrac{\pi}{2}\cos(\pi) = \pi - \tfrac{\pi}{2} = \frac{\pi}{2}$$

Problem 2. (5 points) Set up (but do NOT evaluate) $\iint_S xyz dS$, where S is the cone parameterized by $\mathbf{r}(u,v) = \langle u\cos v, u\sin v, u\rangle$ for $0 \le u \le 1, 0 \le v \le \pi/2$.

$\hat{r}_u = \langle \cos v, \sin v, 1\rangle$, $\hat{r}_v = \langle -u\sin v, u\cos v, 0\rangle$

$$\hat{r}_u \times \hat{r}_v = \begin{vmatrix} i & j & k \\ \cos v & \sin v & 1 \\ -u\sin v & u\cos v & 0 \end{vmatrix} = \langle -u\cos v, -u\sin v, u\rangle$$

$$\Rightarrow \|\hat{r}_u \times \hat{r}_v\| = \sqrt{u^2(\cos^2 v + \sin^2 v) + u^2} = \sqrt{2u^2} = u\sqrt{2}.$$

Then $\iint_S f\,dS = \iint_D f(\hat{r}(u,v))\,\|\hat{r}_u \times \hat{r}_v\|\, du\,dv$

$$= \boxed{\int_0^{\pi/2}\int_0^1 (u\cos v)(u\sin v)(u)(u\sqrt{2})\,du\,dv}$$