Name: Key
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MAC 2313.9722
Cyr

Quiz 13
You must show all work to receive full credit!!

Problem 1. (5 points) Evaluate $\int_C \mathbf{F}d\mathbf{r}$, where $\mathbf{F} = \langle ye^x + \sin y, e^x + x\cos y\rangle$ and C is parameterized by $\mathbf{r}(t) = \left\langle \frac{\sqrt{\tan(t)}}{e^{t-\pi/4}}, 2\cos(2t) \right\rangle$ for $\pi/4 \le t \le \pi$.

$\frac{\partial F_1}{\partial y} = e^x + \cos y = \frac{\partial F_2}{\partial x}$, so $\hat{F}$ is conservative.

$\int (ye^x + \sin y)\,dx = ye^x + x\sin y + f(y)$, $\int (e^x + x\cos y)\,dy = ye^x + x\sin y + g(x)$

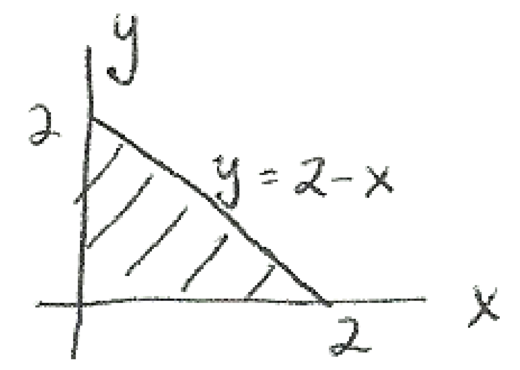
$\Rightarrow f(x,y) = ye^x + x\sin y$ is a potential function for $\hat{F}$. Thus, by the Fundamental Thrm we have

$\int_C \hat{F}\,d\hat{r} = f(\hat{r}(\pi)) - f(\hat{r}(\frac{\pi}{4})) = f(0,2) - f(1,0) = 2e^0 - \sin 0 = 2$

Problem 2. (5 points) Set up (but do NOT evaluate) $\iint_S xzdS$, where S is the portion of the plane $2x + 2y + z = 4$ that lies in the first octant.

$z = 4 - 2x - 2y \Rightarrow z_x = -2,\ z_y = -2$, so $\sqrt{1 + z_x^2 + z_y^2} = \sqrt{1+4+4} = \sqrt{9} = 3$.

Intersection with xy-plane: $2y = 4 - 2x \Rightarrow y = 2 - x$



Then $\iint_S xz\,dS = \iint_D f(x,y)\sqrt{1 + z_x^2 + z_y^2}\,dA$

$= \boxed{\int_0^2 \int_0^{2-x} x(4 - 2x - 2y)\cdot 3\,dy\,dx}$