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MAC 2313.9728
Cyr

Quiz 13
You must show all work to receive full credit!!

**Problem 1.** (5 points) Evaluate $\int_C \mathbf{F} d\mathbf{r}$, where $\mathbf{F} = \langle yze^{xz}, e^{xz}, xye^{xz}\rangle$ and $C$ is parameterized by $\mathbf{r}(t) = \langle t^2+1, t^2-1, t^2-2t\rangle$ for $0 \le t \le 2$.

$$\frac{\partial F_1}{\partial y} = ze^{xz} = \frac{\partial F_2}{\partial x}, \quad \frac{\partial F_2}{\partial z} = xe^{xz} = \frac{\partial F_3}{\partial y}, \quad \frac{\partial F_3}{\partial x} = ye^{xz} + xyze^{xz} = \frac{\partial F_1}{\partial z},$$

So $\hat{F}$ is conservative.

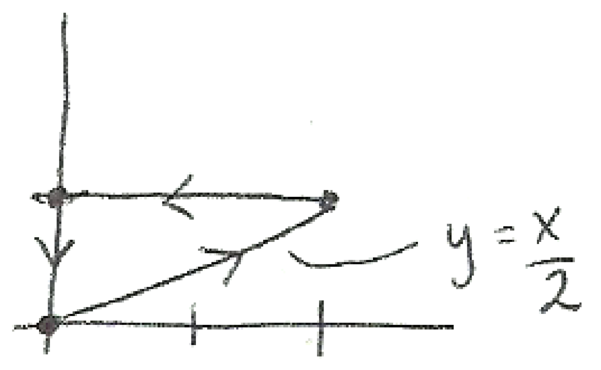
$$\int yze^{xz}\,dx = ye^{xz} + f(y,z)$$
$$\int e^{xz}\,dy = ye^{xz} + g(x,z)$$
$$\int xye^{xz}\,dz = ye^{xz} + h(x,y)$$

$\Rightarrow$ $f(x,y,z) = ye^{xz}$ is a potential function for $\hat{F}$.

Thus, by the Fundamental Thm we have

$$\int_C \hat{F}d\hat{r} = f(\hat{r}(2)) - f(\hat{r}(0)) = f(5,3,0) - f(1,-1,0) = 3-(-1) = 4$$

**Problem 2.** (5 points) Evaluate $\int_C (x^2+y^2)dx + (x^2-y^2)dy$, where $C$ is the triangle with vertices $(0,0)$, $(2,1)$, and $(0,1)$, oriented counterclockwise.




Since we are integrating a two-dimensional vector field over a closed curve, we can apply Green's Theorem.

$$g_x - f_y = 2x - 2y \Rightarrow \int_C \hat{F}d\hat{r} = \int_0^2 \int_{\frac{x}{2}}^1 (2x-2y)\,dy\,dx$$

$$= \int_0^2 2xy - y^2 \Big|_{\frac{x}{2}}^1 dx = \int_0^2 \left[(2x-1) - (x^2 - \tfrac{1}{4}x^2)\right]dx$$

$$= \int_0^2 (2x - 1 - \tfrac{3}{4}x^2)dx = x^2 - x - \tfrac{1}{4}x^3 \Big|_0^2$$

$$= 4 - 2 - 2 = 0$$