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### Quiz 2

You must show all work to receive full credit!!

**Problem 1.** (3 points) Find an explicit general solution to the differential equation

$$x \frac{dy}{dx} + 2y = x^{-3}.$$

Standard form:  $\frac{dy}{dx} + \frac{2}{x}y = x^{-4}$

Integrating factor:  $\mu(x) = e^{\int P(x)dx} = e^{\int (2/x)dx} = e^{2 \ln x} = x^2$

Multiply through by IF:  $x^2 \frac{dy}{dx} + 2xy = x^{-2}$

$$\Rightarrow \int \frac{d}{dx} [x^2 y] = \int x^{-2} dx$$

$$\Rightarrow x^2 y = -x^{-1} + C$$

$$\Rightarrow \boxed{y = -x^{-3} + Cx^{-2}}$$

**Problem 2.** (2 points) Show that the following equation is exact, and then find an implicit solution:

$$(ye^{xy} - y^{-1})dx + (xe^{xy} + xy^{-2})dy = 0.$$

$$M_y = \frac{\partial}{\partial y} [ye^{xy} - y^{-1}] = e^{xy} + xy e^{xy} + y^{-2} \Rightarrow \text{Exact}$$

$$N_x = \frac{\partial}{\partial x} [xe^{xy} + xy^{-2}] = e^{xy} + xy e^{xy} + y^{-2}$$

$$\int M dx = \int (ye^{xy} - y^{-1}) dx = e^{xy} - xy^{-1} + g(y)$$

$$\int N dy = \int (xe^{xy} + xy^{-2}) dy = e^{xy} - xy^{-1} + h(x)$$

$$\Rightarrow \boxed{e^{xy} - xy^{-1} = C}$$