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Quiz 2

You must give complete, mathematically correct proofs to receive full credit!!

Problem 1. (5 points) Let $G = \left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \mid a \in \mathbb{R} \right\}$. Prove that G is a group under matrix multiplication.

Let $A, B \in G$. Then $AB = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & a+b \\ 0 & 1 \end{bmatrix} \in G$, so the operation is closed.

Matrix multiplication is associative. The identity element in G is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ since $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$.

Given $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \in G$, $A^{-1} = \begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix}$ since $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Since $A^{-1} \in G$, G is a group.

Problem 2. (5 points) Let G be a group of functions from $\mathbb{R}$ to $\mathbb{R}$, where the operation is addition of functions. Prove that $H = \{f \in G \mid f(1) = 0\}$ is a subgroup of G.

The identity in G is $e(x) \equiv 0$ $\forall x \in \mathbb{R}$; $e \in H$ since $e(1) = 0$.

Let $f, g \in H$. Then $(f+g)(1) = f(1) + g(1) = 0 + 0 = 0$, so $f+g \in H$.

Let $f \in H$. Its inverse in G is $-f$, and $(-f)(1) = -f(1) = -0 = 0$, so $-f \in H$. Thus, H is a subgroup by the Subgroup Test.