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MAP 2302.4787
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Quiz 3

You must show all work to receive full credit!!

Problem 1. (2 points) Solve the initial value problem

$$y'' - 2y' + 2y = 0, \quad y(\pi) = e^\pi, \quad y'(\pi) = 0.$$

$$\text{Auxiliary eqn: } r^2 - 2r + 2 = 0 \Rightarrow r = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

$$\text{General solution: } y = c_1 e^t \cos t + c_2 e^t \sin t$$

$$\Rightarrow y' = c_1 e^t \cos t - c_1 e^t \sin t + c_2 e^t \sin t + c_2 e^t \cos t$$

$$\text{I.C.: } y(\pi) = -c_1 e^\pi = e^\pi \Rightarrow -c_1 = 1 \Rightarrow c_1 = -1$$

$$y'(\pi) = -c_1 e^\pi - c_2 e^\pi = 0 \Rightarrow c_2 e^\pi = e^\pi \Rightarrow c_2 = 1$$

$$\text{Final solution: } \boxed{y(t) = -e^t \cos t + e^t \sin t}$$

Problem 2. (3 points) Find a general solution for the differential equation

$$y''' + 4y'' - 11y' + 6y = 0.$$

$$\text{Auxiliary eqn: } r^3 + 4r^2 - 11r + 6 = 0.$$

Since sum of coefficients is 1, try synthetic division with $r=1$:

$$\begin{array}{r|rrrr} 1 & 1 & 4 & -11 & 6 \\ & & 1 & 5 & -6 \\ \hline & 1 & 5 & -6 & 0 \end{array}$$

$$(r-1)(r^2 + 5r - 6) = 0$$

$$(r-1)^2(r+6) = 0$$

$$\Rightarrow r=1 \text{ (mult. 2)}, r=-6$$

$$\text{General solution: } \boxed{y(t) = c_1 e^t + c_2 t e^t + c_3 e^{-6t}}$$