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Quiz 3

You must give complete, mathematically correct proofs to receive full credit!!

Problem 1. (5 points) Consider the permutation $\sigma = (13654)(1524637) \in S_7$.
(a) Write σ as a product of disjoint cycles.
(b) Find $|\sigma|$.
(c) Is $\sigma \in A_7$? Explain why or why not.

(a) $\sigma = (1452)(37)$

(b) $|\sigma| = \text{lcm}(2, 4) = 4$

(c) $\sigma = (14)(45)(52)(37) \in A_7$ because it can be written as an even number of transpositions.

Problem 2. (5 points) Let G be a group of permutations on a set X and let $a \in X$. Prove that $\text{stab}_G(a) = \{\alpha \in G \mid \alpha(a) = a\}$ is a subgroup of G.

Proof Let $e(x) = x \ \forall x \in X$ be the identity permutation. Then $e(a) = a$, so $e \in \text{stab}_G(a)$. Suppose $\alpha, \beta \in \text{stab}_G(a)$. Then $\alpha(a) = a$, $\beta(a) = a$. Thus, $\alpha\beta(a) = \alpha(\beta(a)) = \alpha(a) = a$, so $\alpha\beta \in \text{stab}_G(a)$. Suppose $\alpha \in \text{stab}_G(a)$. Then $\alpha(a) = a \Rightarrow \alpha^{-1}(\alpha(a)) = \alpha^{-1}(a)$ $\Rightarrow a = \alpha^{-1}(a)$, so $\alpha^{-1} \in \text{stab}_G(a)$. Thus, $\text{stab}_G(a) \leq G$. ▨