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MAC 2313.8443
Cyr

Quiz 3
You must show all work to receive full credit!!

Problem 1. (5 pts) Solve the initial value problem $\mathbf{r}'(t) = \langle 3t^2, 4t, 1\rangle$ with $\mathbf{r}(1) = \langle 0, 1, 2\rangle$.

$\vec{r}(t) = \int \vec{r}\,'(t)\,dt = \int \langle 3t^2, 4t, 1\rangle\,dt = \langle t^3, 2t^2, t\rangle + \vec{c}$

$\vec{r}(1) = \langle 1^3, 2\cdot 1^2, 1\rangle + \vec{c} = \langle 1, 2, 1\rangle + \vec{c} = \langle 0, 1, 2\rangle$

$\Rightarrow \vec{c} = \langle 0, 1, 2\rangle - \langle 1, 2, 1\rangle = \langle -1, -1, 1\rangle$

Thus $\boxed{\vec{r}(t) = \langle t^3 - 1, 2t^2 - 1, t + 1\rangle}$.

Problem 2. (5 pts) Calculate the length of the curve $\mathbf{r}(t) = \langle 4\sin t, 3\sin t, 5\cos t\rangle$ over the interval $0 \le t \le \pi$.

$\vec{r}(t) = \langle 4\sin t, 3\sin t, 5\cos t\rangle \Rightarrow \vec{r}\,'(t) = \langle 4\cos t, 3\cos t, -5\sin t\rangle$

$\Rightarrow \|\vec{r}\,'(t)\| = \sqrt{(4\cos t)^2 + (3\cos t)^2 + (-5\sin t)^2} = \sqrt{16\cos^2 t + 9\cos^2 t + 25\sin^2 t}$

$= \sqrt{25\cos^2 t + 25\sin^2 t} = \sqrt{25(\sin^2 t + \cos^2 t)} = \sqrt{25} = 5.$

Then $L = \int_a^b \|\vec{r}\,'(t)\|\,dt = \int_0^{\pi} 5\,dt = 5t\Big|_0^{\pi} = \boxed{5\pi}$.