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MAC 2313.9256
Cyr

Quiz 3
You must show all work to receive full credit!!

Problem 1. (5 pts) Find a parametrization of the tangent line $\mathbf{L}(t)$ to the curve $\mathbf{r}(t) = \langle 1 - t^2, 5t, 2t^3 \rangle$ at the point $t = 2$.

$\vec{r}(t) = \langle 1 - t^2, 5t, 2t^3 \rangle \Rightarrow \vec{r}\,'(t) = \langle -2t, 5, 6t^2 \rangle$

So $\vec{r}(2) = \langle 1 - 2^2, 5 \cdot 2, 2 \cdot 2^3 \rangle = \langle -3, 10, 16 \rangle$ and $\vec{r}\,'(2) = \langle -2 \cdot 2, 5, 6 \cdot 2^2 \rangle = \langle -4, 5, 24 \rangle$

Then $\vec{L}(t) = \vec{r}(2) + t\,\vec{r}\,'(2) = \langle -3, 10, 16 \rangle + t\langle -4, 5, 24 \rangle$

$= \boxed{\langle -3 - 4t, 10 + 5t, 16 + 24t \rangle}$

Problem 2. (5 pts) Calculate the length of the curve $\mathbf{r}(t) = \left\langle t^{3/2} + 2, -3, \frac{4}{3}t^{3/2} \right\rangle$ over the interval $1 \leq t \leq 4$.

$\vec{r}(t) = \langle t^{3/2} + 2, -3, \frac{4}{3}t^{3/2} \rangle \Rightarrow \vec{r}\,'(t) = \langle \frac{3}{2}t^{1/2}, 0, 2t^{1/2} \rangle$

$\Rightarrow \|\vec{r}\,'(t)\| = \sqrt{(\frac{3}{2}t^{1/2})^2 + (2t^{1/2})^2} = \sqrt{\frac{9}{4}t + 4t} = \sqrt{\frac{9+16}{4}t} = \sqrt{\frac{25}{4}t} = \frac{5}{2}t^{1/2}$

Then $L = \int_a^b \|\vec{r}\,'(t)\|\,dt = \int_1^4 \frac{5}{2}t^{1/2}\,dt = \frac{5}{3}t^{3/2}\Big|_1^4$

$= \frac{5}{3}(4^{3/2} - 1^{3/2}) = \frac{5}{3}(8 - 1) = \boxed{\frac{35}{3}}$