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Cyr

Quiz 3

You must show all work to receive full credit!!

Problem 1. (3 pts) State the type of quadric surface of the equation $y = 3x^2$ and describe the trace obtained by intersecting with the plane $z = 27$.

Since there is no z in the equation and the trace in the xy-plane is a parabola, this is a $\boxed{\text{parabolic cylinder}}$.

Intersecting with the plane $z=27$ yields $y=3x^2$, which is a $\boxed{\text{parabola}}$.

Problem 2. (7 pts) Let $\mathbf{r}(t) = \langle 2t^2+1, -1, t^3\rangle$.

(a) Find a parametrization of the tangent line $\mathbf{L}(t)$ at the point $t = 2$.

$\vec{r}\,'(t) = \langle 4t, 0, 3t^2\rangle$, so $\vec{r}(2) = \langle 9, -1, 8\rangle$, $\vec{r}\,'(2) = \langle 8, 0, 12\rangle$.

$$\vec{L}(t) = \vec{r}(2) + t\,\vec{r}\,'(2) = \langle 9,-1,8\rangle + t\langle 8,0,12\rangle$$
$$= \boxed{\langle 9+8t, -1, 8+12t\rangle}$$

(b) Compute the length of the curve $\mathbf{r}(t)$ over the interval $0 \le t \le 1$.

$$L = \int_a^b \|\vec{r}\,'(t)\|\,dt = \int_0^1 \sqrt{16t^2+9t^4}\,dt = \int_0^1 \sqrt{t^2(16+9t^2)}\,dt$$
$$= \int_0^1 t\sqrt{16+9t^2}\,dt \qquad \text{Let } u = 16+9t^2,\ du = 18t\,dt$$
$$= \frac{1}{18}\int_{16}^{25} u^{1/2}\,du = \frac{1}{27}\,u^{3/2}\Big|_{16}^{25} = \frac{1}{27}\left(25^{3/2} - 16^{3/2}\right)$$
$$= \frac{1}{27}(125-64) = \boxed{\frac{61}{27}}$$