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MAC 2313.3122
Cyr

Quiz 3
You must show all work to receive full credit!!

Problem 1. (3 pts) State the type of quadric surface of the equation $(\frac{x}{3})^2+(\frac{y}{5})^2-5z^2=1$ and describe the trace obtained by intersecting with the plane $y=1$.

The equation has the form $\left(\frac{x}{3}\right)^2+\left(\frac{y}{5}\right)^2 = 5z^2+1$, so it is a $\boxed{\text{hyperboloid of one sheet.}}$

Intersecting with the plane $y=1$ gives $\left(\frac{x}{3}\right)^2+\frac{1}{25}-5z^2=1$

$\Rightarrow \left(\frac{x}{3}\right)^2-\left(\frac{z}{1/\sqrt{5}}\right)^2=\frac{24}{25}$, which is a $\boxed{\text{hyperbola}}$.

Problem 2. (7 pts) Let $\mathbf{r}(t)=\langle\cos(3t),\sin(3t),3t\rangle$.
(a) Find a parametrization of the tangent line $\mathbf{L}(t)$ at the point $t=\frac{\pi}{6}$.

$\vec{r}\,'(t)=\langle -3\sin(3t), 3\cos(3t), 3\rangle$, so $\vec{r}(\frac{\pi}{6})=\langle 0, 1, \frac{\pi}{2}\rangle$,

$\vec{r}\,'(\frac{\pi}{6})=\langle -3,0,3\rangle$.

Then $\vec{L}(t)=\vec{r}(\frac{\pi}{6})+t\vec{r}\,'(\frac{\pi}{6})=\langle 0,1,\frac{\pi}{2}\rangle+t\langle -3,0,3\rangle$

$=\boxed{\langle -3t, 1, \frac{\pi}{2}+3t\rangle}$

(b) Compute the length of the curve $\mathbf{r}(t)$ over the interval $0\le t\le 2\pi$.

$$L=\int_a^b \|\vec{r}\,'(t)\|\,dt=\int_0^{2\pi}\sqrt{9\sin^2(3t)+9\cos^2(3t)+9}\,dt$$

$$=\int_0^{2\pi}\sqrt{9(\sin^2(3t)+\cos^2(3t))+9}\,dt=\int_0^{2\pi}\sqrt{18}\,dt$$

$$=3\sqrt{2}\int_0^{2\pi}dt=3\sqrt{2}(2\pi)=\boxed{6\pi\sqrt{2}}$$