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Quiz 3

You must show all work to receive full credit!!

Problem 1. (3 pts) Solve the initial value problem $\mathbf{r}'(t) = \left\langle \frac{t}{\sqrt{t^2+5}}, \frac{1}{t-1}, e^{t-2} \right\rangle$ with $\mathbf{r}(2) = \langle 8, -3, 2 \rangle$.

$$\vec{r}(t) = \left\langle \int \frac{t}{\sqrt{t^2+5}}\,dt,\ \int \frac{dt}{t-1},\ \int e^{t-2}\,dt \right\rangle = \langle \sqrt{t^2+5},\ \ln(t-1),\ e^{t-2} \rangle + \vec{c}$$

$u = t^2+5 \quad \frac{1}{2}du = t\,dt$

$$\vec{r}(2) = \langle \sqrt{4+5},\ \ln(1),\ e^0 \rangle + \vec{c} = \langle 3, 0, 1 \rangle + \vec{c} = \langle 8, -3, 2 \rangle$$

$$\Rightarrow \vec{c} = \langle 5, -3, 1 \rangle$$

$$\boxed{\vec{r}(t) = \langle \sqrt{t^2+5} + 5,\ \ln(t-1) - 3,\ e^{t-2} + 1 \rangle}$$

Problem 2. (4 pts) Find the equation of the tangent line to the graph of $\mathbf{r}(t) = \langle \cos(2t), \sin(2t), 2t \rangle$ at $t = \frac{\pi}{3}$.

$$\vec{r}\,'(t) = \langle -2\sin(2t),\ 2\cos(2t),\ 2 \rangle \Rightarrow \vec{r}\,'\left(\tfrac{\pi}{3}\right) = \left\langle -2\left(\tfrac{\sqrt{3}}{2}\right),\ 2\left(-\tfrac{1}{2}\right),\ 2 \right\rangle$$

$$= \langle -\sqrt{3}, -1, 2 \rangle$$

$$\vec{r}\left(\tfrac{\pi}{3}\right) = \left\langle -\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}, \tfrac{2\pi}{3} \right\rangle$$

Tangent line has equation $\boxed{\vec{L}(t) = \left\langle -\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}, \tfrac{2\pi}{3} \right\rangle + t\langle -\sqrt{3}, -1, 2 \rangle}$

Problem 3. (3 pts) Calculate the arc length of the curve $\mathbf{r}(t) = \langle \cos(2t), \sin(2t), 2t \rangle$ on the interval $0 \le t \le 2\pi$.

From #2, $\vec{r}\,'(t) = \langle -2\sin(2t), 2\cos(2t), 2 \rangle$, so

$$L = \int_0^{2\pi} \|\vec{r}\,'(t)\|\,dt = \int_0^{2\pi} \sqrt{4\sin^2(2t) + 4\cos^2(2t) + 4}\,dt = \int_0^{2\pi} \sqrt{4(\sin^2(2t) + \cos^2(2t) + 1)}\,dt$$

$$= \int_0^{2\pi} \sqrt{4 \cdot 2}\,dt = 2\sqrt{2}t\Big|_0^{2\pi} = \boxed{4\pi\sqrt{2}}$$