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MAC 2313.9722
Cyr

Quiz 3

You must show all work to receive full credit!!

Problem 1. (4 points) Evaluate the integral $\int_0^1 \left\langle \frac{1}{t+1}, \frac{1}{t^2+1}, \frac{t}{t^2+1} \right\rangle dt$.

$$\int \frac{dt}{t+1} = \ln(t+1), \quad \int \frac{dt}{t^2+1} = \tan^{-1}(t), \quad \int \frac{t dt}{t^2+1} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln(t^2+1)$$

$u = t^2+1, du = 2t dt$

$$\Rightarrow \left\langle \ln(t+1), \tan^{-1}(t), \frac{1}{2} \ln(t^2+1) \right\rangle \Big|_0^1$$

$$= \boxed{\left\langle \ln(2), \pi/4, \frac{1}{2} \ln(2) \right\rangle}$$

Problem 2. (6 points) Find the length of the curve $\mathbf{r}(t) = \langle 2t^2 + 1, -1, t^3 \rangle$ over the interval $0 \leq t \leq 1$.

$$\hat{\mathbf{r}}'(t) = \langle 4t, 0, 3t^2 \rangle$$

$$\Rightarrow \|\hat{\mathbf{r}}'(t)\| = \sqrt{16t^2 + 9t^4} = \sqrt{t^2(16 + 9t^2)} = t\sqrt{16 + 9t^2} \quad (\text{no abs. value needed since } t \geq 0)$$

$$L = \int_a^b \|\hat{\mathbf{r}}'(t)\| dt = \int_0^1 t\sqrt{16 + 9t^2} dt \quad u = 9t^2 + 16 \quad du = 18t dt$$

$$= \frac{1}{18} \int_{16}^{25} u^{1/2} du = \frac{1}{27} u^{3/2} \Big|_{16}^{25}$$

$$= \frac{1}{27} (25^{3/2} - 16^{3/2}) = \frac{1}{27} (125 - 64) = \boxed{\frac{61}{27}}$$