Name: Key
July 20, 2016
MAP 2302.4787
Cyr

Quiz 4
You must show all work to receive full credit!!

Problem 1. (2 points) Write the form of a particular solution suggested by the method of undetermined coefficients. DO NOT find the coefficients.

(a) $y'' + 2y' = t^2 + t$

Aux eqn: $r^2 + 2r = 0$

$r(r+2) = 0$

$\Rightarrow r = 0$ is a simple root

$t^2 + t = (t^2 + t)e^{0t}$

Guess $\boxed{y_p = t(At^2 + Bt + C)}$

(b) $y'' - 4y' + 4y = te^{2t} - \sin(2t)$

Aux eqn.: $r^2 - 4r + 4 = 0$

$(r-2)^2 = 0$

$\Rightarrow r = 2$ is a double root

Guess $\boxed{y_p = t^2(At + B)e^{2t} + C\sin(2t) + D\cos(2t)}$

Problem 2. (3 points) Use variation of parameters to find a general solution to the differential equation

$$y'' + 2y' + y = e^{-t}.$$

Aux eqn: $r^2 + 2r + 1 = 0 \Rightarrow (r+1)^2 = 0 \Rightarrow r = -1$ (mult. 2)

$y_h = c_1e^{-t} + c_2te^{-t}$. Let $y_1 = e^{-t}$, $y_2 = te^{-t}$.

$$\left| \begin{array}{l} e^{-t}v_1' + te^{-t}v_2' = 0 \\ -e^{-t}v_1' + (e^{-t} - te^{-t})v_2' = e^{-t} \end{array} \right.$$

Adding equations gives $e^{-t}v_2' = e^{-t} \Rightarrow v_2' = 1 \Rightarrow v_2 = t$.

Substitute $v_2' = 1$: $e^{-t}v_1' = -te^{-t} \Rightarrow v_1' = -t \Rightarrow v_1 = -\frac{1}{2}t^2$.

Then $y_p = v_1y_1 + v_2y_2 = -\frac{1}{2}t^2e^{-t} + t^2e^{-t} = \frac{1}{2}t^2e^{-t}$ and

general solution is $\boxed{y(t) = c_1e^{-t} + c_2te^{-t} + \frac{1}{2}t^2e^{-t}}$