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Quiz 4

You must give complete, mathematically correct proofs to receive full credit!!

Problem 1. (5 points) Let $G = (\mathbb{R}^+, \cdot)$ and $H = (\mathbb{R}, +)$. Show that $\phi : G \to H$ defined by $\phi(x) = \ln x$ is an isomorphism.

Proof Let $x, y \in \mathbb{R}^+$ and assume that $\phi(x) = \phi(y)$. Then $\ln x = \ln y \Rightarrow e^{\ln x} = e^{\ln y} \Rightarrow x = y$, so ϕ is injective. Let $y \in \mathbb{R}$. Then $e^y \in \mathbb{R}^+$ and $\phi(e^y) = \ln e^y = y$, so ϕ is surjective.

To show that ϕ preserves the group operations, let $x, y \in \mathbb{R}^+$.

Then $\phi(xy) = \ln(xy) = \ln x + \ln y = \phi(x) + \phi(y)$.

Thus, ϕ is an isomorphism. ▨

Problem 2. (5 points) Let $G = S_3$. Find the image of every element of G under the inner automorphism of G induced by the element (12).

Recall that $\phi_{(12)}(x) = (12)\,x\,(12)^{-1} = (12)\,x\,(12)$ $\forall x \in S_3$.

So we have
$$\phi_{(12)}(1) = (12)(1)(12) = (1)$$
$$\phi_{(12)}(12) = (12)(12)(12) = (12)$$
$$\phi_{(12)}(13) = (12)(13)(12) = (23)$$
$$\phi_{(12)}(23) = (12)(23)(12) = (13)$$
$$\phi_{(12)}(123) = (12)(123)(12) = (132)$$
$$\phi_{(12)}(132) = (12)(132)(12) = (123)$$