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MAC 2312.5828
Cyr

Quiz 4

You must show all work to receive full credit!!

Problem 1. (3 pts) Determine whether the sequence $a_n = (1+\frac{6}{n})^{2n}$ converges or diverges; if it converges, find the limit.

Consider $\lim_{n\to\infty} 2n \ln(1+\frac{6}{n}) = \lim_{n\to\infty} 2 \cdot \frac{\ln(1+\frac{6}{n})}{\frac{1}{n}}$

$\overset{(LH)}{=} 2 \lim_{n\to\infty} \dfrac{\dfrac{-\frac{6}{n^2}}{1+\frac{6}{n}}}{\dfrac{-1}{n^2}} = 2 \lim_{n\to\infty} \dfrac{6}{1+\frac{6}{n}} = 2 \cdot 6 = 12,$

So $\lim_{n\to\infty} (1+\frac{6}{n})^{2n} = \boxed{e^{12}}$.

Problem 2. (2 pts) Determine whether the series $\sum_{n=1}^{\infty} \ln(\frac{n^2+1}{8n^2+5})$ converges or diverges. If it is convergent, find the sum.

Since $\lim_{n\to\infty} \ln\left(\frac{n^2+1}{8n^2+5}\right) = \ln\left(\frac{1}{8}\right) \neq 0$, the series

$\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{8n^2+5}\right)$ $\boxed{\text{diverges}}$ by the nth term divergence test.