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MAC 2313.3122
Cyr

Quiz 4

You must show all work to receive full credit!!

Problem 1. Let $f(x, y) = \frac{1}{x^2 + y^2 + 1}$.

(a) (2 pts) Evaluate $f(-3, 5)$.

$$f(-3, 5) = \frac{1}{(-3)^2 + 5^2 + 1} = \frac{1}{9 + 25 + 1} = \boxed{\frac{1}{35}}$$

(b) (3 pts) Find the domain of $f(x, y)$.

We want $x^2 + y^2 + 1 \neq 0$, but $x^2 + y^2 + 1 \geq 1$ for all $(x, y) \in \mathbb{R}^2$,

$$\text{So Domain} = \mathbb{R}^2$$

(c) (2 pts) Find the range of $f(x, y)$ (write your answer in interval notation).

Since $x^2 + y^2 + 1 > 0$ for all $(x, y) \in \mathbb{R}^2$, certainly $\frac{1}{x^2 + y^2 + 1} > 0$.

Also, $x^2 + y^2 + 1 \geq 1$ for all $(x, y) \in \mathbb{R}^2$ implies $\frac{1}{x^2 + y^2 + 1} \leq 1$,

$$\text{So Range} = (0, 1].$$

(d) (3 pts) Write an equation for the level curve $f(x, y) = \frac{1}{5}$, and use it to describe the graph of the level curve.

$$\frac{1}{x^2 + y^2 + 1} = \frac{1}{5} \Rightarrow x^2 + y^2 + 1 = 5 \Rightarrow \boxed{x^2 + y^2 = 4}$$

This is a circle centered at the origin of radius 2.