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MAC 2313.8326
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Quiz 4
You must show all work to receive full credit!!

Problem 1. Let $f(x,y) = \sqrt{9-(x^2+y^2)}$.
(a) (2 pts) Find the domain of $f(x,y)$.

Want $9-(x^2+y^2) \geq 0 \Rightarrow x^2+y^2 \leq 9$, so

D: $\{(x,y) \mid x^2+y^2 \leq 9\}$

(b) (2 pts) Find the range of $f(x,y)$ (write your answer in interval notation).

Certainly $\sqrt{9-(x^2+y^2)} \geq 0$, max value occurs when x^2+y^2 is minimum.
Since $x^2+y^2 \geq 0$, $9-(x^2+y^2) \leq 9 \Rightarrow \sqrt{9-(x^2+y^2)} \leq \sqrt{9} = 3$. Thus,

R: $[0, 3]$

(c) (2 pts) Write an equation for the level curve $f(x,y) = \sqrt{5}$, and use it to describe the graph of the level curve.

$\sqrt{5} = \sqrt{9-(x^2+y^2)} \Rightarrow 5 = 9-(x^2+y^2) \Rightarrow \boxed{x^2+y^2 = 4}$

The level curve is a circle of radius 2 centered at the origin.

Problem 2. Let $g(x,y) = \dfrac{x^3y}{x^4+y^3}$.
(a) (1 pt) Evaluate $\lim\limits_{(x,y)\to(0,0)} g(x,y)$ along the line $y = x$.

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^4+y^3} = \lim_{x\to 0} \frac{x^4}{x^4+x^3} = \lim_{x\to 0} \frac{x^4}{x^3(x+1)} = \lim_{x\to 0} \frac{x}{x+1} = \frac{0}{1} = \boxed{0}$$

(b) (1 pt) Evaluate $\lim_{(x,y)\to(0,0)} g(x,y)$ along the curve $y = x^2$.

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^4+y^3} = \lim_{x\to 0} \frac{x^5}{x^4+x^6} = \lim_{x\to 0} \frac{x^5}{x^4(x^2+1)} = \lim_{x\to 0} \frac{x}{x^2+1} = \frac{0}{1} = 0$$

(c) (2 pts) What can you conclude about $\lim_{(x,y)\to(0,0)} g(x,y)$? Explain your answer.

Nothing - even though these paths have the same limit, there could be another path with a different limit, causing the limit not to exist.