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Quiz 5

You must show all work to receive full credit!!

Problem 1. (2 pts) Find the sum of the series $\sum_{n=2}^{\infty} \frac{4}{n(n+2)}$.

$$\frac{4}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2} \qquad A = 2 \quad B = -2$$

$$= 2\sum_{n=2}^{\infty}\left(\frac{1}{n} - \frac{1}{n+2}\right) = 2\sum_{n=2}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1} + \frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$S_N = 2\Big[\frac{1}{2} - \cancel{\frac{1}{3}} + \frac{1}{3} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} \; \vdots \; \cancel{\frac{1}{N}} - \frac{1}{N+1} + \cancel{\frac{1}{N+1}} - \frac{1}{N+2}\Big] = 2\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{N+1} - \frac{1}{N+2}\right)$$

$$\lim_{N\to\infty} S_N = 1 + \frac{2}{3} = \frac{5}{3}$$

Problem 2. (3 pts) Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty} \frac{n}{e^{6n}}$ converges or diverges. (Be sure to justify why the Integral Test applies.)

Since $f(x) = \frac{x}{e^{6x}}$ is positive + continuous for $x \geq 1$ and

$f'(x) = \frac{e^{6x}(1-6x)}{(e^{6x})^2} = \frac{1-6x}{e^{6x}} < 0$ for $x > 1/6$, f is also decreasing, so

apply the integral test: $\int_1^{\infty} \frac{x}{e^{6x}}\,dx$

IBP

x +	e^{-6x}
1 −	$-\frac{1}{6}e^{-6x}$
0	$\frac{1}{36}e^{-6x}$

$$= -\frac{1}{6}xe^{-6x} - \frac{1}{36}e^{-6x}\Big|_1^{\infty}$$

$$= \lim_{R\to\infty}\left(\frac{-R}{6e^{6R}} - \frac{1}{36e^{6R}} + \frac{1}{6e^6} + \frac{1}{36e^6}\right)$$

$$= \frac{1}{6e^6} + \frac{1}{36e^6} < \infty,$$

so $\sum_{n=1}^{\infty} \frac{n}{e^{6n}}$ $\boxed{\text{Converges}}$.