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Quiz 5
You must show all work to receive full credit!!

Problem 1. (2 pts) Find the sum of the series $\sum_{n=1}^{\infty}(e^{1/n} - e^{1/(n+1)})$.

$$S_N = e^1 - \cancel{e^{1/2}}$$
$$\cancel{e^{1/2}} - \cancel{e^{1/3}}$$
$$\vdots$$
$$+ \cancel{e^{1/N}} - e^{1/N+1}$$
$$e - e^{1/N+1}$$

$$\sum_{n=1}^{\infty}(e^{1/n} - e^{1/(n+1)}) = \lim_{N\to\infty} S_N = \lim_{N\to\infty}(e - e^{1/N+1}) = \boxed{e-1}$$

Problem 2. (3 pts) Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty}\frac{n^2}{e^{9n}}$ converges or diverges. (Be sure to justify why the Integral Test applies.)

The function $f(x) = \frac{x^2}{e^{9x}}$ is certainly positive + continuous for $x \geq 1$.

Since $f'(x) = \frac{e^{9x}(2x - 9x^2)}{(e^{9x})^2} = \frac{x(2-9x)}{e^{9x}} < 0$ when $x > \frac{2}{9}$, f is also decreasing.

So apply integral test: $\int_1^{\infty} \frac{x^2}{e^{9x}}\,dx$

IBP

x^2	$+\ e^{-9x}$
$2x$	$-\ -\frac{1}{9}e^{-9x}$
2	$+\ \frac{1}{81}e^{-9x}$
0	$-\frac{1}{729}e^{-9x}$

$$= \lim_{R\to\infty} -\frac{1}{9}x^2e^{-9x} - \frac{2}{81}xe^{-9x} - \frac{2}{729}e^{-9x}\Big|_1^R$$

$$= \lim_{R\to\infty} \frac{-R^2}{9e^{9R}} - \frac{2R}{81e^{9R}} - \frac{2}{729e^{9R}} + \frac{1}{9e^9} + \frac{2}{81e^9} + \frac{2}{729e^9}$$

$$\overset{LH}{=} \frac{1}{9e^9} + \frac{2}{81e^9} + \frac{2}{729e^9} < \infty, \text{ so } \sum_{n=1}^{\infty}\frac{n^2}{e^{9n}}\ \boxed{\text{converges}}.$$