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October 14, 2014
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Quiz 5

You must show all work to receive full credit!!

Problem 1. (2 pts) Find the sum of the series $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$.

$$\frac{2}{(n-1)(n+1)} = \frac{A}{n-1} + \frac{B}{n+1} \qquad A = 1 \quad B = -1$$

$$\sum_{n=2}^{\infty}\left(\frac{1}{n-1} - \frac{1}{n+1}\right) = \sum_{n=2}^{\infty}\left(\frac{1}{n-1} - \frac{1}{n} + \frac{1}{n} - \frac{1}{n+1}\right)$$

$$S_N = 1 - \cancel{\tfrac{1}{2}} + \tfrac{1}{2} - \cancel{\tfrac{1}{3}}$$
$$\cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{3}} + \cancel{\tfrac{1}{3}} - \cancel{\tfrac{1}{4}}$$
$$\vdots$$
$$\cancel{\tfrac{1}{N-1}} - \tfrac{1}{N} + \cancel{\tfrac{1}{N}} - \tfrac{1}{N+1}$$
$$= \frac{3}{2} - \frac{1}{N} - \frac{1}{N+1}$$

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \lim_{N\to\infty}\left(\frac{3}{2} - \frac{1}{N} - \frac{1}{N+1}\right) = \frac{3}{2}$$

Problem 2. (3 pts) Use the Integral Test to determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{3\ln n}{n^5}$ converges or diverges. (Be sure to justify why the Integral Test applies.)

Since $f(x) = \dfrac{3\ln x}{x^5}$ is positive & continuous for $x \geq 1$ and

$f'(x) = 3\left(\dfrac{x^4 - 5x^4 \ln x}{x^{10}}\right) = 3\left(\dfrac{1-5\ln x}{x^6}\right) < 0$ for $\ln x > 1/5 \Rightarrow x > e^{1/5} \approx 1.22$,

f is decreasing for $x \geq 2$. Apply the integral test:

$$3\int_2^{\infty} \frac{\ln x}{x^5}\,dx \qquad u = \ln x \quad dv = x^{-5}dx \qquad du = \frac{dx}{x} \quad v = -\frac{x^{-4}}{4}$$

$$= 3\left[-\frac{\ln x}{4x^4} + \frac{1}{4}\int x^{-5}dx\right] = -\frac{3\ln x}{4x^4} - \frac{3}{16x^4}\Bigg|_2^{\infty}$$

$$= \lim_{R\to\infty}\left(-\frac{3\ln R}{4R^4} - \frac{3}{16R^4} + \frac{3\ln 2}{64} + \frac{3}{256}\right) = \frac{3\ln 2}{64} + \frac{3}{256} < \infty$$

Hence $\displaystyle\sum_{n=2}^{\infty}\frac{3\ln n}{n^5}$ converges, and $\displaystyle\sum_{n=1}^{\infty}\frac{3\ln n}{n^5}$ also $\boxed{\text{converges}}$.