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Quiz 5
You must show all work to receive full credit!!

Problem 1. (6 pts) Let $f(x,y) = \dfrac{xy}{x-y}$. Evaluate $\left.\dfrac{\partial f}{\partial y}\right|_{(2,3)}$ and $\left.\dfrac{\partial^2 f}{\partial x \partial y}\right|_{(4,2)}$.

$$\frac{\partial f}{\partial y} = \frac{(x-y)\cdot x - (xy)\cdot(-1)}{(x-y)^2} = \frac{x^2 - xy + xy}{(x-y)^2} = \frac{x^2}{(x-y)^2}$$

$$\Rightarrow \left.\frac{\partial f}{\partial y}\right|_{(2,3)} = \frac{2^2}{(2-3)^2} = \frac{4}{(-1)^2} = \boxed{4}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left[\frac{\partial f}{\partial y}\right] = \frac{(x-y)^2 \cdot 2x - x^2 \cdot 2(x-y)}{(x-y)^4} = \frac{2x(x-y)[(x-y) - x]}{(x-y)^4}$$

$$= \frac{-2xy}{(x-y)^3} \Rightarrow \left.\frac{\partial^2 f}{\partial x \partial y}\right|_{(4,2)} = \frac{-2(4)(2)}{(4-2)^3} = \frac{-16}{8} = \boxed{-2}$$

Problem 2. (4 pts) Let $g(x,y) = \dfrac{x^2y^2}{x^2+y^2}$.

(a) Evaluate $\lim\limits_{(x,y)\to(0,0)} g(x,y)$ along the x-axis.

Along x-axis, $y=0$, so $\lim\limits_{x\to 0} \dfrac{x^2\cdot 0^2}{x^2+0^2} = \lim\limits_{x\to 0} \dfrac{0}{x^2}$

$= \lim\limits_{x\to 0} 0 = \boxed{0}$

(b) Evaluate $\lim\limits_{(x,y)\to(0,0)} g(x,y)$ along the line $y = x$.

$$\lim_{x\to 0} \frac{x^2\cdot x^2}{x^2+x^2} = \lim_{x\to 0} \frac{x^4}{2x^2} = \lim_{x\to 0} \frac{1}{2}x^2 = \boxed{0}$$

(c) Can you conclude that $\lim\limits_{(x,y)\to(0,0)} g(x,y)$ exists? Explain why or why not.

No: even though the limit along two paths is equal, there could be another path which has a different limit.