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MAC 2313.8326
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Quiz 5
You must show all work to receive full credit!!

Problem 1. (4 pts) Let $f(x,y) = \dfrac{x}{x+4y}$. Evaluate $\left.\dfrac{\partial f}{\partial y}\right|_{(2,-1)}$ and $\left.\dfrac{\partial^2 f}{\partial y^2}\right|_{(2,-1)}$.

$$\frac{\partial f}{\partial y} = \frac{(x+4y)(0) - x(4)}{(x+4y)^2} = \frac{-4x}{(x+4y)^2}\Bigg|_{(2,-1)} = \frac{-4(2)}{(2-4)^2} = \frac{-8}{4} = \boxed{-2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left[\frac{\partial f}{\partial y}\right] = \frac{(x+4y)^2(0) - (-4x)[2(x+4y)4]}{(x+4y)^4} = \frac{32x(x+4y)}{(x+4y)^4}$$

$$= \frac{32x}{(x+4y)^3}\Bigg|_{(2,-1)} = \frac{64}{(2-4)^3} = \frac{64}{-8} = \boxed{-8}$$

Problem 2. Let $f(x,y,z) = \dfrac{xy}{z}$. $= xyz^{-1}$

(a) (5 pts) Find the linearization $L(x,y,z)$ of f at the point $(2,1,2)$.

$$f_x = yz^{-1} = \frac{y}{z} \qquad f_y = xz^{-1} = \frac{x}{z} \qquad f_z = -xyz^{-2} = \frac{-xy}{z^2}$$

$$f_x(2,1,2) = \frac{1}{2} \qquad f_y(2,1,2) = 1 \qquad f_z(2,1,2) = \frac{-2}{4} = -\frac{1}{2}$$

$$L(x,y,z) = f_x(x-2) + f_y(y-1) + f_z(z-2) + f(2,1,2)$$

$$= \boxed{\frac{1}{2}(x-2) + (y-1) - \frac{1}{2}(z-2) + 1}$$

$$= \frac{1}{2}x + y - \frac{1}{2}z$$

(b) (1 pt) Use your linearization to approximate the value $\dfrac{(2.05)(0.9)}{2.01}$.

$$f(2.05, 0.9, 2.01) \approx L(2.05, 0.9, 2.01) = \frac{1}{2}(2.05-2) + (0.9-1) - \frac{1}{2}(2.01-2) + 1$$

$$= \frac{1}{2}(.05) - 0.1 - \frac{1}{2}(.01) + 1 = \frac{1}{2}(.04) + 0.9 = .02 + 0.9 = \boxed{0.92}$$