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MAC 2312.0703
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Quiz 6

You must show all work to receive full credit!!

Problem 1. (2.5 pts) Determine whether the series $\sum_{n=1}^{\infty} \frac{2+n+2n^2}{\sqrt{1+n^2+n^6}}$ converges or diverges. (Be sure to state which test(s) you are using.)

Let $b_n = \frac{n^2}{\sqrt{n^6}} = \frac{n^2}{n^3} = \frac{1}{n}$ and note $\sum b_n = \sum \frac{1}{n}$ diverges (harmonic series).

Then $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{2+n+2n^2}{\sqrt{1+n^2+n^6}} \cdot \frac{\sqrt{n^6}}{n^2} = 2\sqrt{\lim_{n\to\infty} \frac{n^6}{1+n^2+n^6}} = 2.$

Since $0 < 2 < \infty$, the series $\sum_{n=1}^{\infty} \frac{2+n+2n^2}{\sqrt{1+n^2+n^6}}$ also $\boxed{\text{diverges}}$ by the Limit Comparison Test.

Problem 2. (2.5 pts) Determine whether the series $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{3/5}}$ is absolutely convergent, conditionally convergent, or divergent. (Be sure to state which test(s) you are using.)

Note $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{3/5}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/5}}$. Since $\lim_{n\to\infty} \frac{1}{n^{3/5}} = 0$ and

$\frac{1}{(n+1)^{3/5}} < \frac{1}{n^{3/5}}$, the series converges by the Alternating Series Test.

Considering $\sum |a_n|$, $\sum_{n=1}^{\infty} \frac{1}{n^{3/5}}$ is a divergent p-series ($p = 3/5 < 1$).

Thus, the series $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{3/5}}$ is $\boxed{\text{conditionally convergent.}}$