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Quiz 6
You must show all work to receive full credit!!

Problem 1. (2.5 points) Find a general solution to the system

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & 3 \\ 12 & 1 \end{bmatrix} \mathbf{x}(t).$$

Char. polyn. $p(r) = \begin{vmatrix} 1-r & 3 \\ 12 & 1-r \end{vmatrix} = r^2 - 2r - 35 = 0 \Rightarrow (r-7)(r+5) = 0$

$\Rightarrow$ eigenvalues are $r=7$, $r=-5$

$r=7$: $\begin{bmatrix} -6 & 3 \\ 12 & -6 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \hat{0} \Rightarrow \begin{matrix} 3u_2 = 6u_1 \\ u_2 = 2u_1 \end{matrix} \Rightarrow \hat{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$r=-5$: $\begin{bmatrix} 6 & 3 \\ 12 & 6 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \hat{0} \Rightarrow \begin{matrix} 3u_2 = -6u_1 \\ u_2 = -2u_1 \end{matrix} \Rightarrow \hat{u}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

General solution: $\hat{x}(t) = c_1 e^{7t}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{-5t}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Problem 2. (2.5 points) Given that the vectors below are solutions to the system $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$, show that they form a fundamental solution set, and write a fundamental matrix for the system.

$$\mathbf{x}_1 = \begin{bmatrix} e^t \\ e^t \\ e^t \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} \sin t \\ \cos t \\ -\sin t \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} -\cos t \\ \sin t \\ \cos t \end{bmatrix}$$

$W[x_1, x_2, x_3] = \begin{vmatrix} e^t & \sin t & -\cos t \\ e^t & \cos t & \sin t \\ e^t & -\sin t & \cos t \end{vmatrix} = e^t(\cos^2 t + \sin^2 t) - \sin t\,(e^t \cos t - e^t \sin t) - \cos t\,(-e^t \sin t - e^t \cos t)$

$= e^t - e^t \sin t \cos t + e^t(\sin^2 t + \cos^2 t) + e^t \sin t \cos t$

$= \boxed{2e^t}$

Since $2e^t \neq 0$ for all t, the vectors are lin. indep. + form a fundamental solution set. The fundamental matrix is $\hat{X}(t) = \begin{bmatrix} e^t & \sin t & -\cos t \\ e^t & \cos t & \sin t \\ e^t & -\sin t & \cos t \end{bmatrix}$.