Name: Key
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MAC 2312.5885
Cyr

Quiz 6

You must show all work to receive full credit!!

Problem 1. (2.5 pts) Determine whether the series $\sum_{n=5}^{\infty} \frac{6\sqrt{n}}{n-4}$ converges or diverges. (Be sure to state which test(s) you are using.)

Note that $\sum_{n=5}^{\infty} \frac{6\sqrt{n}}{n-4} \geq \sum_{n=5}^{\infty} \frac{6\sqrt{n}}{n} = 6\sum_{n=5}^{\infty} \frac{1}{\sqrt{n}}$. Since $6\sum_{n=5}^{\infty} \frac{1}{\sqrt{n}}$ is a divergent p-series $(p=\frac{1}{2}<1)$, $\sum_{n=5}^{\infty} \frac{6\sqrt{n}}{n-4}$ also $\boxed{\text{diverges}}$ by (direct) comparison test.

OR, let $b_n = \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}}$ and note $\sum b_n = \sum \frac{1}{\sqrt{n}}$ is a divergent p-series $(p=\frac{1}{2}<1)$.

Then $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{6\sqrt{n}}{n-4} \cdot \frac{n}{\sqrt{n}} = 6\lim_{n\to\infty} \frac{n}{n-4} = 6$, and since $0<6<\infty$,

$\sum_{n=5}^{\infty} \frac{6\sqrt{n}}{n-4}$ $\boxed{\text{diverges}}$ also by Limit Comparison Test.

Problem 2. (2.5 pts) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-12)^n}{(n+1)8^{2n+1}}$ is absolutely convergent, conditionally convergent, or divergent. (Be sure to state which test(s) you are using.)

Apply the Ratio Test:

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{12^{n+1}\,(n+1)\,8^{2n+1}}{12^n\,(n+2)\,8^{2n+3}} = \frac{12}{8^2}\lim_{n\to\infty} \frac{n+1}{n+2} = \frac{12}{64}.$$

Since $\frac{12}{64}<1$, $\sum_{n=1}^{\infty} \frac{(-12)^n}{(n+1)8^{2n+1}}$ is $\boxed{\text{absolutely convergent}}$ by the Ratio Test.

OR, $\sum_{n=1}^{\infty} \left|\frac{(-12)^n}{(n+1)8^{2n+1}}\right| \leq \sum_{n=1}^{\infty} \frac{12^n}{(8^2)^n \cdot 8} = \frac{1}{8}\sum_{n=1}^{\infty} \left(\frac{12}{64}\right)^n$. Since $\frac{1}{8}\sum_{n=1}^{\infty} \left(\frac{12}{64}\right)^n$ is a convergent geometric series $(r=\frac{12}{64}<1)$, $\sum_{n=1}^{\infty} \frac{(-12)^n}{(n+1)8^{2n+1}}$ is $\boxed{\text{absolutely convergent}}$ by the (direct) comparison test.