Name: Key
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MAC 2313.8443
Cyr

Quiz 6
You must show all work to receive full credit!!

Problem 1. (6 pts) Let $f(x,y) = xy^2 - xy + 3x^3y$.
(a) Find an equation of the tangent plane to $f(x,y)$ at the point $(1,3)$.

$f_x = y^2 - y + 9x^2y\,|_{(1,3)} = 3^2 - 3 + 9\cdot 1^2\cdot 3 = 9-3+27 = \underline{33}$

$f_y = 2xy - x + 3x^3\,|_{(1,3)} = 2\cdot 1\cdot 3 - 1 + 3\cdot 1^3 = 6 - 1 + 3 = \underline{8}$

$f(1,3) = 1\cdot 3^2 - 1\cdot 3 + 3\cdot 1^3\cdot 3 = 9 - 3 + 9 = 15$, so

$z = f(1,3) + f_x(1,3)(x-1) + f_y(1,3)(y-3)$

$\Rightarrow \boxed{z = 15 + 33(x-1) + 8(y-3)}$ or $33x + 8y - z = 42$

(b) Calculate the directional derivative of $f(x,y)$ in the direction of $\mathbf{v} = \langle -1, 5\rangle$ at the point $(1,3)$.

$D_{\vec{v}} f(1,3) = \dfrac{\nabla f(1,3)\cdot \vec{v}}{\|\vec{v}\|} = \dfrac{\langle 33, 8\rangle \cdot \langle -1, 5\rangle}{\sqrt{26}} = \dfrac{-33+40}{\sqrt{26}} = \boxed{\dfrac{7}{\sqrt{26}}}$

$\nabla f(1,3) = \langle 33, 8\rangle$ from part (a)

$\|\vec{v}\| = \sqrt{(-1)^2 + 5^2} = \sqrt{1+25} = \sqrt{26}$

Problem 2. (4 pts) Let $g(x,y) = (x-y)e^x$, $x = u - v$, $y = u + v$. Use the chain rule to evaluate the partial derivative $\dfrac{\partial g}{\partial v}$ at the point $(x,y) = (1,4)$.

$\dfrac{\partial g}{\partial v} = \dfrac{\partial g}{\partial x}\cdot\dfrac{\partial x}{\partial v} + \dfrac{\partial g}{\partial y}\cdot\dfrac{\partial y}{\partial v} = \left(e^x + (x-y)e^x\right)(-1) + (-e^x)(1)$

$= -e^x - (x-y)e^x - e^x = -2e^x - (x-y)e^x$

$\Rightarrow \dfrac{\partial g}{\partial v}\Big|_{(1,4)} = -2e^1 - (1-4)e^1 = -2e + (+3)e = \boxed{e}$