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February 18, 2016
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Quiz 6

You must show all work to receive full credit!!

Problem 1. (6 pts) Let $f(x,y) = \dfrac{x^2}{y^2+1}$. $= x^2(y^2+1)^{-1}$

(a) Find an equation of the tangent plane to $f(x,y)$ at the point $(4,1)$.

$$f_x = 2x(y^2+1)^{-1} = \frac{2x}{y^2+1}\Big|_{(4,1)} = \frac{2\cdot 4}{1^2+1} = \frac{8}{2} = \underline{4}$$

$$f_y = -1x^2(y^2+1)^{-2}(2y) = \frac{-2x^2y}{(y^2+1)^2}\Big|_{(4,1)} = \frac{-2\cdot 4^2\cdot 1}{(1+1)^2} = \frac{-32}{4} = \underline{-8}$$

$$f(4,1) = \frac{4^2}{1^2+1} = \frac{16}{2} = 8, \text{ so } L(x,y) = z = f(4,1) + f_x(4,1)(x-4) + f_y(4,1)(y-1)$$

$$\Rightarrow \boxed{z = 8 + 4(x-4) - 8(y-1)} \quad \text{or} \quad 4x - 8y - z = 0$$

(b) Calculate the directional derivative of $f(x,y)$ in the direction of $\mathbf{v} = \langle 3,2\rangle$ at the point $(4,1)$.

$$D_{\vec{v}}f(4,1) = \frac{\nabla f(4,1)\cdot\vec{v}}{\|\vec{v}\|} = \frac{\langle 4,-8\rangle\cdot\langle 3,2\rangle}{\sqrt{13}} = \frac{12-16}{\sqrt{13}} = \boxed{\frac{-4}{\sqrt{13}}}$$

$\nabla f(4,1) = \langle 4,-8\rangle$ by part (a)

$\|\vec{v}\| = \sqrt{3^2+2^2} = \sqrt{9+4} = \sqrt{13}$

Problem 2. (4 pts) Let $g(x,y) = x^2 - y^2$, $x = e^u\cos(v)$, $y = e^u\sin(v)$. Use the chain rule to evaluate the partial derivative $\dfrac{\partial g}{\partial u}$ at the point $(u,v) = (0,\pi)$.

$$\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial g}{\partial y}\cdot\frac{\partial y}{\partial u} = (2x)(e^u\cos v) + (-2y)(e^u\sin v)$$

$$= 2(e^u\cos v)^2 - 2(e^u\sin v)^2$$

$$\Rightarrow \frac{\partial g}{\partial u}\Big|_{(0,\pi)} = 2(e^0\cos\pi)^2 - \cancel{2(e^0\sin\pi)^2}^{\,0} = 2(-1)^2 = \boxed{2}$$