Name: Key
February 19, 2015
MAC 2313.3118
Cyr

Quiz 6
You must show all work to receive full credit!!

Problem 1. (6 pts) Let $f(x, y) = x^2 + y^{-2}$.
(a) Find an equation of the tangent plane to $f(x, y)$ at the point $(4, 1)$.

$f_x(x,y) = 2x \Rightarrow f_x(4,1) = 2(4) = 8$

$f_y(x,y) = -2y^{-3} = \frac{-2}{y^3} \Rightarrow f_y(4,1) = \frac{-2}{1} = -2$

$f(4,1) = 4^2 + \frac{1}{1^2} = 16 + 1 = 17$

So $z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$\boxed{z = 17 + 8(x-4) - 2(y-1)}$

OR $z = 8x - 2y - 13$

(b) Calculate the directional derivative of $f(x, y)$ in the direction of $\mathbf{v} = \langle 1, 2\rangle$ at the point $(4, 1)$.

$D_{\vec{v}} f(4,1) = \frac{1}{\|\vec{v}\|}\left(\nabla f(4,1)\cdot\vec{v}\right) = \frac{\langle 8,-2\rangle\cdot\langle 1,2\rangle}{\sqrt{5}} = \frac{8-4}{\sqrt{5}} = \boxed{\frac{4}{\sqrt{5}}}$

$\nabla f(4,1) = \langle 8,-2\rangle$ by part (a)

$\|\vec{v}\| = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$

Problem 2. (4 pts) Let $g(x, y) = \ln(xy)$, $x = 3r + 2s$, $y = 5r + 3s$. Use the chain rule to evaluate the partial derivative $\frac{\partial g}{\partial s}$ at the point $(r, s) = (1, 0)$.

$\frac{\partial g}{\partial s} = \frac{\partial g}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial g}{\partial y}\cdot\frac{\partial y}{\partial s} = \frac{1}{x}\cdot 2 + \frac{1}{y}\cdot 3 = \frac{2}{x} + \frac{3}{y}$

$(r,s) = (1,0) \Rightarrow (x,y) = (3,5)$, so

$\left.\frac{\partial g}{\partial s}\right|_{(r,s)=(1,0)} = \frac{2}{3} + \frac{3}{5} = \frac{10+9}{15} = \boxed{\frac{19}{15}}$