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MAC 2313.3122
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Quiz 6

You must show all work to receive full credit!!

Problem 1. (6 pts) Let $f(x, y) = x^2y^{-1/2} + y^{-3}$.
(a) Find an equation of the tangent plane to $f(x, y)$ at the point $(2, 1)$.

$f_x(x,y) = 2xy^{-1/2} \Rightarrow f_x(2,1) = \frac{2(2)}{\sqrt{1}} = 4$

$f_y(x,y) = -\frac{1}{2}x^2y^{-3/2} - 3y^{-4} \Rightarrow f_y(2,1) = \frac{-\frac{1}{2}(2)^2}{1^{3/2}} - \frac{3}{1^4} = -2-3 = -5$

$f(2,1) = \frac{2^2}{\sqrt{1}} + \frac{1}{1^3} = 4+1 = 5$

So $z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$\Rightarrow \boxed{z = 5 + 4(x-2) - 5(y-1)}$

OR $z = 4x - 5y + 2$

(b) Calculate the directional derivative of $f(x, y)$ in the direction of $\mathbf{v} = \langle 3, 4\rangle$ at the point $(2, 1)$.

$D_{\vec{v}} f(2,1) = \frac{1}{\|\vec{v}\|}(\nabla f(2,1) \cdot \vec{v}) = \frac{1}{5}(\langle 4,-5\rangle \cdot \langle 3,4\rangle) = \frac{12-20}{5} = \boxed{\frac{-8}{5}}$

$\nabla f(2,1) = \langle 4, -5\rangle$ from part (a)

$\|\vec{v}\| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

Problem 2. (4 pts) Let $g(x, y) = xe^y$, $x = s^3$, $y = sr^2$. Use the chain rule to evaluate the partial derivative $\dfrac{\partial g}{\partial s}$ at the point $(r, s) = (2, 2)$.

$\frac{\partial g}{\partial s} = \frac{\partial g}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial g}{\partial y}\cdot\frac{\partial y}{\partial s} = e^y\cdot(3s^2) + xe^y\cdot r^2$

$(r,s) = (2,2) \Rightarrow (x,y) = (8,8)$, so

$\left.\frac{\partial g}{\partial s}\right|_{(r,s)=(2,2)} = e^8(3\cdot 2^2) + 8e^8\cdot 2^2 = 12e^8 + 32e^8 = \boxed{44e^8}$