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Quiz 6
You must show all work to receive full credit!!

Problem 1. Let $f(x, y, z) = xe^{-yz}$ and $\mathbf{v} = \langle 1, 1, 1\rangle$.
(a) (4 pts) Find $D_{\mathbf{v}}f(1, 2, 0)$.

$\nabla f = \langle f_x, f_y, f_z \rangle = \langle e^{-yz}, -xze^{-yz}, -xye^{-yz} \rangle$

$\nabla f(1,2,0) = \langle e^0, 0, -2e^0 \rangle = \langle 1, 0, -2 \rangle$

$\|\vec{v}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$D_v f(1,2,0) = \nabla f(1,2,0) \cdot \hat{v} = \langle 1,0,-2 \rangle \cdot \dfrac{\langle 1,1,1 \rangle}{\sqrt{3}} = \dfrac{1-2}{\sqrt{3}} = \boxed{\dfrac{-1}{\sqrt{3}}}$

(b) (2 pts) Find the equation of the tangent plane to the level surface $xe^{-yz} = 1$ at the point $(1, 2, 0)$.

A normal vector for the tangent plane is $\nabla f(1,2,0) = \langle 1, 0, -2 \rangle$,
so $\langle x - x_0, y - y_0, z - z_0 \rangle \cdot \langle a, b, c \rangle = 0$

$\Rightarrow \boxed{\langle x-1, y-2, z \rangle \cdot \langle 1, 0, -2 \rangle = 0}$

or $x - 2z = 1$

Problem 2. (4 pts) Let $g(x, y) = x^2 - y^2$, $x = e^u \cos v$, $y = e^u \sin v$. Find $\dfrac{\partial g}{\partial u}$ at the point $(u, v) = (0, \pi)$.

$$\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial g}{\partial y} \cdot \frac{\partial y}{\partial u}$$
$$= (2x)(e^u \cos v) + (-2y)(e^u \sin v)$$
$$= 2(e^u \cos v)^2 - 2(e^u \sin v)^2$$

$$\left.\frac{\partial g}{\partial u}\right|_{(u,v)=(0,\pi)} = 2(e^0 \cos \pi)^2 - 2(e^0 \sin \pi)^2 = 2(-1)^2 = \boxed{2}$$