Name: Key
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MAC 2312.0703
Cyr

Quiz 7

You must show all work to receive full credit!!

Problem 1. (2 pts) Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-10)^n}{n^2+1}$.

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x-10|^{n+1}(n^2+1)}{|x-10|^n((n+1)^2+1)} = |x-10| \lim_{n\to\infty} \frac{n^2+1}{n^2+2n+2} = |x-10| < 1$

$\Rightarrow$ ROC = 1

At $x=9$, $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$ converges by AST since $\lim_{n\to\infty} \frac{1}{n^2+1} = 0$ and $\frac{1}{(n+1)^2+1} < \frac{1}{n^2+1}$

At $x=11$, $\sum_{n=0}^{\infty} \frac{1}{n^2+1} \le \sum_{n=0}^{\infty} \frac{1}{n^2}$ a convergent p-series ($p=2>1$), so converges by comparison test.

Hence, IOC = $[9, 11]$

Problem 2. (3 pts) Find a power series representation for the function $f(x) = \frac{x}{49+x^2}$ and determine the interval of convergence.

$f(x) = \frac{x}{49} \cdot \frac{1}{1+\frac{x^2}{49}} = \frac{x}{49} \sum_{n=0}^{\infty} \left(\frac{-x^2}{49}\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{49^{n+1}}$

$\left|\frac{x^2}{49}\right| < 1 \Rightarrow |x|^2 < 49 \Rightarrow |x| < 7$, so IOC = $(-7, 7)$