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Quiz 7

You must show all work to receive full credit!!

Problem 1. (2.5 pts) Find the radius and interval of convergence of the series $\sum_{n=3}^{\infty} \frac{x^{n+11}}{\sqrt{n}}$.

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x|^{n+12}}{|x|^{n+11}} \frac{\sqrt{n}}{\sqrt{n+1}} = |x| \sqrt{\lim_{n\to\infty} \frac{n}{n+1}} = |x| < 1 \Rightarrow \boxed{ROC = 1}$

At $x = -1$, $\sum_{n=3}^{\infty} \frac{(-1)^{n+11}}{\sqrt{n}}$ converges by AST since $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ and $\frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}}$.

At $x = 1$, $\sum_{n=3}^{\infty} \frac{1}{\sqrt{n}}$ is a divergent p-series $(p = \frac{1}{2} < 1)$.

So IOC: $\boxed{[-1, 1)}$

Problem 2. (2.5 pts) Find a power series representation for the function $f(x) = \dfrac{x}{5x^2 + 1}$ and determine the interval of convergence.

Note $\frac{1}{5x^2+1} = \frac{1}{1-(-5x^2)} = \sum_{n=0}^{\infty} (-5x^2)^n = \sum_{n=0}^{\infty} (-1)^n 5^n x^{2n}$.

Then $f(x) = x \cdot \frac{1}{5x^2+1} = \boxed{\sum_{n=0}^{\infty} (-1)^n 5^n x^{2n+1}}$.

This converges for $|5x^2| < 1 \Rightarrow |x|^2 < \frac{1}{5} \Rightarrow |x| < \frac{1}{\sqrt{5}}$, so

IOC: $\boxed{(-1/\sqrt{5},\ 1/\sqrt{5})}$