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Quiz 7

You must show all work to receive full credit!!

Problem 1. (2.5 pts) Find the radius and interval of convergence of the series $\sum_{n=2}^{\infty}(-1)^n\frac{x^n}{11^n \ln n}$.

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{|x|^{n+1}\,11^n \ln n}{|x|^n\,11^{n+1}\ln(n+1)} = \frac{|x|}{11}\lim_{n\to\infty}\frac{\ln(n)}{\ln(n+1)} \overset{(LH)}{=} \frac{|x|}{11}\lim_{n\to\infty}\frac{n+1}{n} = \frac{|x|}{11} < 1$

$\Rightarrow |x| < 11 \Rightarrow$ **ROC = 11**

At $x = -11$, $\sum_{n=2}^{\infty}\frac{(-1)^n(-1)^n 11^n}{11^n \ln n} = \sum_{n=2}^{\infty}\frac{1}{\ln n} \geq \sum_{n=2}^{\infty}\frac{1}{n}$ - divergent harmonic series, so diverges by (direct) comparison test.

At $x = 11$, $\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n}$ converges by AST since $\lim_{n\to\infty}\frac{1}{\ln n} = 0$ and $\frac{1}{\ln(n+1)} < \frac{1}{\ln n}$.

So IOC: **(-11, 11]**

Problem 2. (2.5 pts) Find a power series representation for the function $f(x) = \dfrac{9}{4-x}$ and determine the interval of convergence.

$f(x) = \frac{9}{4-x} = \frac{9}{4}\cdot\frac{1}{1-\frac{x}{4}} = \frac{9}{4}\sum_{n=0}^{\infty}\frac{x^n}{4^n} = \boxed{\sum_{n=0}^{\infty}\frac{9x^n}{4^{n+1}}}$

$\left|\frac{x}{4}\right| < 1 \Rightarrow |x| < 4$, so IOC: **(-4, 4)**