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Quiz 7
You must show all work to receive full credit!!

Problem 1. (3 pts) Evaluate the iterated integral $\int_1^2 \int_0^2 (x^2 - y^2)\,dy\,dx$.

$$\int_1^2 \left[x^2y - \frac{1}{3}y^3 \Big|_0^2 \right] dx = \int_1^2 \left(2x^2 - \frac{8}{3}\right) dx$$

$$= \frac{2}{3}x^3 - \frac{8}{3}x \Big|_1^2 = \left[\frac{16}{3} - \frac{16}{3}\right] - \left[\frac{2}{3} - \frac{8}{3}\right]$$

$$= 0 + \left(+\frac{6}{3}\right) = \boxed{2}$$

Problem 2. (3 pts) Find and classify the critical points of $f(x, y) = \ln x + 2\ln y - x - 4y$.

$f_x = \frac{1}{x} - 1 = 0 \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1$

$f_y = \frac{2}{y} - 4 = 0 \Rightarrow \frac{2}{y} = 4 \Rightarrow y = \frac{1}{2}$

, so critical point is $(1, \frac{1}{2})$

$f_{xx} = \frac{-1}{x^2}$

$f_{yy} = \frac{-2}{y^2}$

$f_{xy} = 0$

$D = f_{xx}f_{yy} - f_{xy}^2 = \frac{2}{x^2y^2} - 0$

$D(1, \frac{1}{2}) = \frac{2}{1 \cdot 1/4} = 8 > 0$

$f_{xx}(1, \frac{1}{2}) = -1 < 0$

$\Rightarrow (1, \frac{1}{2})$ is a $\boxed{\text{local max}}$

Problem 3. (4 pts) Use the method of Lagrange multipliers to find the minimum and maximum values of the function $f(x, y) = xy$ subject to the constraint $4x^2 + 9y^2 = 32$.

$g(x,y) = 4x^2 + 9y^2 - 32$

$\nabla f = \langle y, x \rangle \quad \nabla g = \langle 8x, 18y \rangle \Rightarrow y = \lambda 8x$, $x = \lambda 18y$ (note $x, y \neq 0$)

$\Rightarrow \lambda = \frac{y}{8x} = \frac{x}{18y} \Rightarrow 8x^2 = 18y^2 \Rightarrow y^2 = \frac{4}{9}x^2 \Rightarrow y = \pm\frac{2}{3}x$

$y = \frac{2}{3}x \Rightarrow 4x^2 + 9(\frac{4}{9}x^2) = 32 \Rightarrow 8x^2 = 32 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$, so $(2, \frac{4}{3}), (-2, -\frac{4}{3})$

$y = -\frac{2}{3}x \Rightarrow x = \pm 2$, so $(2, -\frac{4}{3}), (-2, \frac{4}{3})$

$f(2, \frac{4}{3}) = f(-2, -\frac{4}{3}) = \boxed{\frac{8}{3} : \text{max}}$ $\quad f(2, -\frac{4}{3}) = f(-2, \frac{4}{3}) = \boxed{\frac{-8}{3} : \text{min}}$