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Quiz 7
You must show all work to receive full credit!!

Problem 1. (3 points) Find the maximum rate of change of the function $f(x, y, z) = \dfrac{x}{y+z}$ at the point $(8, 1, 3)$.

Max rate of change at $(8,1,3)$ is $\|\nabla f(8,1,3)\|$.

$\nabla f = \left\langle \frac{1}{y+z}, \frac{-x}{(y+z)^2}, \frac{-x}{(y+z)^2} \right\rangle \Rightarrow \nabla f(8,1,3) = \left\langle \frac{1}{4}, -\frac{1}{2}, -\frac{1}{2} \right\rangle$

So $\|\nabla f(8,1,3)\| = \sqrt{\frac{1}{16}+\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{9}{16}} = \boxed{\frac{3}{4}}$

Problem 2. (7 points) Find and classify the critical points of the function $f(x, y) = x - x^2y - y + xy^2$. (Hint: Consider $f_x + f_y$.)

$f_x = 1 - 2xy + y^2 = 0$
$f_y = -x^2 - 1 + 2xy = 0$

$\Rightarrow f_x + f_y = y^2 - x^2 = 0 \Rightarrow y^2 = x^2 \Rightarrow y = \pm x$

If $y = x$ in $f_x = 0$: $1 - 2x^2 + x^2 = 1 - x^2 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

If $y = -x$ in $f_x = 0$: $1 + 2x^2 + x^2 = 3x^2 + 1 = 0 \Rightarrow x^2 = -\frac{1}{3}$ - no soln.

Critical points: $(1,1), (-1,-1)$

$f_{xx} = -2y$, $f_{yy} = 2x$, $f_{xy} = -2x + 2y$, so

$D = f_{xx} f_{yy} - f_{xy}^2 = -4xy - (2y - 2x)^2$.

$D(1,1) = -4 - 0^2 = -4 < 0 \Rightarrow$ $(1,1)$ is a saddle point

$D(-1,-1) = -4 - 0^2 = -4 < 0 \Rightarrow$ $(-1,-1)$ is a saddle point