Name: Key
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MAC 2313.9728
Cyr

Quiz 7
You must show all work to receive full credit!!

Problem 1. (4 points) Find the directional derivative of the function $f(x,y,z) = xy^2 \arctan z$ at the point $(2,1,1)$ in the direction of the vector $\mathbf{v} = \langle 4,-1,1\rangle$.

$\nabla f = \langle y^2 \arctan z,\ 2xy \arctan z,\ \frac{xy^2}{1+z^2} \rangle$

$\Rightarrow \nabla f(2,1,1) = \langle 1 \cdot \frac{\pi}{4},\ 2\cdot 2\cdot 1\cdot \frac{\pi}{4},\ \frac{2\cdot 1^2}{1+1^2} \rangle = \langle \frac{\pi}{4}, \pi, 1 \rangle$

$\|\hat{v}\| = \sqrt{16+1+1} = \sqrt{18} = 3\sqrt{2}$, so $\hat{u} = \frac{\langle 4,-1,1\rangle}{3\sqrt{2}}$.

$D_{\hat{v}} f(2,1,1) = \nabla f(2,1,1) \cdot \hat{u} = \frac{\langle \pi/4, \pi, 1\rangle \cdot \langle 4,-1,1\rangle}{3\sqrt{2}} = \frac{\pi - \pi + 1}{3\sqrt{2}}$

$= \boxed{\frac{1}{3\sqrt{2}}}$ (or $\frac{\sqrt{2}}{6}$)

Problem 2. (6 points) Find and classify the critical points of the function

$$f(x,y) = x^2 + y^4 + 2xy.$$

$f_x = 2x + 2y = 0 \Rightarrow 2y = -2x \Rightarrow y = -x$

$f_y = 4y^3 + 2x = 0 \Rightarrow 4(-x)^3 + 2x = -4x^3 + 2x = 0$

$\Rightarrow 2x(-2x^2+1) = 0 \Rightarrow x = 0$ or $x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

Critical points: $(0,0)$, $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$, $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$f_{xx} = 2$, $f_{yy} = 12y^2$, $f_{xy} = 2$, so $D = f_{xx}f_{yy} - f_{xy}^2 = 24y^2 - 4$.

$D(0,0) = -4 < 0 \Rightarrow$ $\boxed{(0,0) \text{ is a saddle point}}$

$D(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 8 > 0$ and $f_{xx} = 2 > 0 \Rightarrow$ $\boxed{(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) \text{ is a local min}}$

$D(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 8 > 0$ and $f_{xx} = 2 > 0 \Rightarrow$ $\boxed{(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \text{ is a local min}}$