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Quiz 8

You must show all work to receive full credit!!

Problem 1. (2 pts) Find the Taylor series for $f(x) = x^4 - 6x^2 + 1$ centered at $c = 2$.

n	$f^{(n)}(x)$	$f^{(n)}(2)$	$c_n = \frac{f^{(n)}(2)}{n!}$
0	x^4-6x^2+1	$16-24+1 = -7$	-7
1	$4x^3 - 12x$	$32-24 = 8$	8
2	$12x^2 - 12$	$48-12 = 36$	$36/2 = 18$
3	$24x$	48	$48/6 = 8$
4	24	24	$24/24 = 1$
5	0	0	0
	⋮	⋮	⋮

$$f(x) = \sum_{n=0}^{\infty} c_n (x-2)^n$$

$$f(x) = -7 + 8(x-2) + 18(x-2)^2 + 8(x-2)^3 + (x-2)^4$$

Problem 2. (3 pts) Consider $x = 3\cos\theta$, $y = \sin 2\theta$.

(a) Find the points (x, y) on the curve where the tangent is horizontal.

Since $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$, tangent is horizontal when $\frac{dy}{d\theta} = 0$:

$\frac{dy}{d\theta} = 2\cos 2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

At $\theta = \frac{\pi}{4}$, $x = \frac{3}{\sqrt{2}}$, $y = 1$ $\left(\frac{3}{\sqrt{2}}, 1\right)$ At $\theta = \frac{3\pi}{4}$, $\left(-\frac{3}{\sqrt{2}}, -1\right)$

At $\theta = \frac{5\pi}{4}$, $\left(-\frac{3}{\sqrt{2}}, 1\right)$ At $\theta = \frac{7\pi}{4}$, $\left(\frac{3}{\sqrt{2}}, -1\right)$

(b) Find the points (x, y) on the curve where the tangent is vertical.

Tangent is vertical when $\frac{dx}{d\theta} = 0$ ($\frac{dy}{dx}$ is undefined):

$\frac{dx}{d\theta} = -3\sin\theta = 0 \Rightarrow \theta = 0, \pi$

At $\theta = 0$, $(3, 0)$ At $\theta = \pi$, $(-3, 0)$