Name: Key
March 10, 2016
MAC 2313.8443
Cyr

Quiz 8
You must show all work to receive full credit!!

Problem 1. (4 pts) Evaluate the iterated integral $\int_{-1}^{2}\int_{0}^{\pi} x\sin(xy)dydx$.

$$\int_{-1}^{2}\int_{0}^{\pi} x\sin(xy)\,dy\,dx = \int_{-1}^{2} -\cos(xy)\Big|_{0=y}^{\pi}\,dx$$

$$= \int_{-1}^{2}\left[-\cos(\pi x) + (+\cos 0)\right]dx = \int_{-1}^{2}\left(1-\cos(\pi x)\right)dx$$

$$= x - \frac{\sin(\pi x)}{\pi}\Big|_{-1}^{2} = \left(2 - \frac{\sin(2\pi)}{\pi}\right) - \left(-1 - \frac{\sin(-\pi)}{\pi}\right)$$

$$= 2 + (+1) = \boxed{3}$$

Problem 2. (6 pts) Use the method of Lagrange multipliers to find the minimum and maximum values of the function $f(x,y) = 6x + 8y$ subject to the constraint $x^2 + y^2 = 25$.

$\nabla f = \langle 6, 8\rangle$, $\nabla g = \langle 2x, 2y\rangle$ so $\nabla f = \lambda \nabla g$ implies

$$\begin{cases} 6 = 2\lambda x \\ 8 = 2\lambda y \end{cases} \Rightarrow \lambda = \frac{3}{x} = \frac{4}{y} \Rightarrow 3y = 4x \Rightarrow y = \frac{4}{3}x$$

(OK since $x \neq 0, y \neq 0$)

Sub into constraint: $x^2 + \left(\frac{4}{3}x\right)^2 = 25 \Rightarrow x^2 + \frac{16}{9}x^2 = 25$

$\Rightarrow \frac{25}{9}x^2 = 25 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$, so critical points are $(3, 4)$ and $(-3, -4)$.

$f(3,4) = 6(3) + 8(4) = 18 + 32 = \boxed{50}$ max

$f(-3,-4) = 6(-3) + 8(-4) = -18 - 32 = \boxed{-50}$ min