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MAC 2313.3118
Cyr

Quiz 8
You must show all work to receive full credit!!

Problem 1. (4 pts) Evaluate the iterated integral $\int_2^6 \int_1^4 x^2 dx dy$.

$$\int_1^4 x^2 dx = \frac{x^3}{3}\Big|_1^4 = \frac{64}{3} - \frac{1}{3} = \frac{63}{3} = 21$$

$$\Rightarrow \int_2^6 \int_1^4 x^2 dx\, dy = \int_2^6 21\, dy = 21y\Big|_2^6 = 21(6-2)$$

$$= 21 \cdot 4 = \boxed{84}$$

Problem 2. (6 pts) Use the method of Lagrange multipliers to find the minimum and maximum values of the function $f(x,y) = xy$ subject to the constraint $4x^2 + 9y^2 = 32$.

$\nabla f = \langle y, x \rangle$ $\quad g(x,y) = 4x^2 + 9y^2 - 32 \Rightarrow \nabla g = \langle 8x, 18y \rangle$

$\nabla f = \lambda \nabla g \Rightarrow \begin{cases} y = 8x\lambda \\ x = 18y\lambda \end{cases} \Rightarrow \lambda = \frac{y}{8x} = \frac{x}{18y} \Rightarrow 18y^2 = 8x^2$

(can divide since $(0,0)$ is not on constraint curve)

$\Rightarrow y^2 = \frac{4}{9}x^2 \Rightarrow y = \pm\frac{2}{3}x$

Sub into constraint: $4x^2 + 9\left(\frac{4}{9}x^2\right) = 32 \Rightarrow 4x^2 + 4x^2 = 8x^2 = 32 \Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$, so critical points are $(2, \frac{4}{3}), (2, -\frac{4}{3}), (-2, -\frac{4}{3}), (-2, \frac{4}{3})$.

$f(2, \frac{4}{3}) = \frac{8}{3}$ - maximum

$f(2, -\frac{4}{3}) = \frac{-8}{3}$ - minimum