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MAC 2313.3122
Cyr

Quiz 8

You must show all work to receive full credit!!

Problem 1. (4 pts) Evaluate the iterated integral $\int_2^6 \int_1^4 y^2 dx dy$.

$$\int_1^4 y^2 dx = y^2 x \Big|_{x=1}^4 = y^2(4-1) = 3y^2, \text{ so}$$

$$\int_2^6 \int_1^4 y^2 dx dy = \int_2^6 3y^2 dy = y^3 \Big|_2^6 = 216 - 8 = \boxed{208}$$

Problem 2. (6 pts) Use the method of Lagrange multipliers to find the minimum and maximum values of the function $f(x, y) = 2x + 4y$ subject to the constraint $x^2 + y^2 = 5$.

$$\nabla f = \langle 2, 4 \rangle \quad g(x, y) = x^2 + y^2 - 5 \Rightarrow \nabla g = \langle 2x, 2y \rangle.$$

$$\text{Then } \nabla f = \lambda \nabla g \Rightarrow \begin{cases} 2 = 2x\lambda \\ 4 = 2y\lambda \end{cases} \Rightarrow \lambda = \frac{1}{x} = \frac{2}{y} \Rightarrow y = 2x$$

(can divide since $(0,0)$ not on constraint curve)

$$\text{Sub. into constraint: } x^2 + (2x)^2 = 5 \Rightarrow x^2 + 4x^2 = 5 \Rightarrow 5x^2 = 5 \\ \Rightarrow x^2 = 1 \Rightarrow x = \pm 1, \text{ so critical points are } (1, 2), (-1, -2).$$

$$f(1, 2) = 2 + 8 = \boxed{10 : \text{maximum}}$$

$$f(-1, -2) = -2 - 8 = \boxed{-10 : \text{minimum}}$$