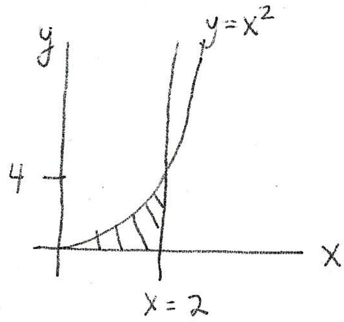
Name: Key
July 9, 2015
MAC 2313.8326
Cyr

Quiz 8
You must show all work to receive full credit!!

Problem 1. (3 pts) Evaluate by first changing the order of integration: $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1}\, dx dy.$

(You may find it helpful to sketch the domain of integration.)



$$\int_0^2 \int_0^{x^2} \sqrt{x^3+1}\, dy dx = \int_0^2 y\sqrt{x^3+1}\Big|_{y=0}^{x^2} dx$$

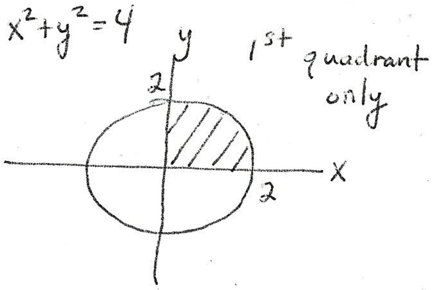
$$= \int_0^2 x^2\sqrt{x^3+1}\, dx \qquad u = x^3+1 \quad du = 3x^2 dx$$

$$= \frac{1}{3}\int_1^9 u^{1/2} du = \frac{2}{9} u^{3/2}\Big|_1^9 = \frac{2}{9}(27-1) = \frac{2}{9}(26)$$

$$= \boxed{\frac{52}{9}}$$

Problem 2. (4 pts) Set up the triple integral that would be used to calculate the volume of the region in the first octant ($x \geq 0, y \geq 0, z \geq 0$) above $z = y^2$ and below $z = 8-2x^2-y^2$. DO NOT EVALUATE.

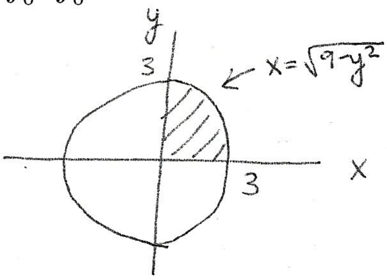
Find curve of intersection: $y^2 = 8-2x^2-y^2 \Rightarrow 2x^2+2y^2 = 8 \Rightarrow x^2+y^2=4$



$$\boxed{V = \int_0^2 \int_0^{\sqrt{4-x^2}} \int_{y^2}^{8-2x^2-y^2} dz\, dy\, dx}$$

Problem 3. (3 pts) Evaluate by first changing to polar coordinates: $\int_0^3 \int_0^{\sqrt{9-y^2}} \sqrt{x^2+y^2}\, dx dy.$

(You may find it helpful to sketch the domain of integration.)



$$= \int_0^{\pi/2}\int_0^3 \sqrt{r^2}\, r dr d\theta = \int_0^{\pi/2}\int_0^3 r^2 dr d\theta$$

$$= \int_0^{\pi/2} \frac{r^3}{3}\Big|_0^3 d\theta = \int_0^{\pi/2} 9\, d\theta = 9\theta\Big|_0^{\pi/2} = \boxed{\frac{9\pi}{2}}$$