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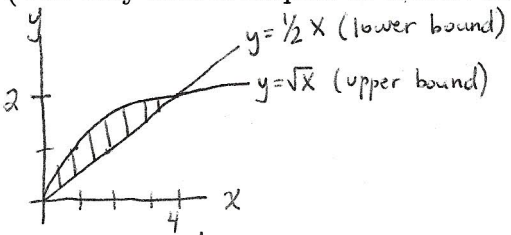
MAC 2313.8443
Cyr

Quiz 9
You must show all work to receive full credit!!

Problem 1. (2 pts) Rewrite the integral by changing the order of integration: $\int_0^2 \int_{y^2}^{2y} f(x,y)\,dxdy$.
(You may find it helpful to sketch the domain of integration.)



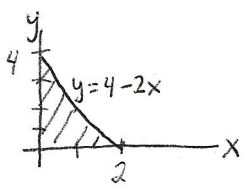
$x=2y \Rightarrow y=\frac{1}{2}x$
$x=y^2 \Rightarrow y=\sqrt{x}$

$\Rightarrow \boxed{\int_0^4 \int_{\frac{x}{2}}^{\sqrt{x}} f(x,y)\,dydx}$

Problem 2. (3 pts) Set up the triple integral that would be used to calculate the volume of the region in the first octant ($x \geq 0, y \geq 0, z \geq 0$) satisfying $2x+y+z \leq 4$. DO NOT EVALUATE. (You may find it helpful to sketch the domain of integration.)

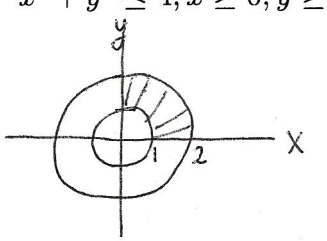
$0 \leq z \leq 4-2x-y$

In the x,y-plane ($z=0$), we have $2x+y \leq 4 \Rightarrow y \leq 4-2x$.



$\boxed{\int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} dzdydx}$

Problem 3. (5 pts) Evaluate by using polar coordinates: $\iint_D x\,dA$, where $\mathcal{D} = \{1 \leq x^2+y^2 \leq 4, x \geq 0, y \geq 0\}$. (You may find it helpful to sketch the domain of integration.)



$1 \leq r \leq 2$
$0 \leq \theta \leq \frac{\pi}{2}$

$$\int_0^{\pi/2} \int_1^2 r\cos\theta \cdot r\,dr\,d\theta = \int_0^{\pi/2} \int_1^2 r^2\cos\theta\,dr\,d\theta$$
$$= \int_0^{\pi/2} \frac{r^3}{3}\Big|_1^2 \cos\theta\,d\theta = \frac{7}{3}\int_0^{\pi/2} \cos\theta\,d\theta$$
$$= \frac{7}{3}\sin\theta\Big|_0^{\pi/2} = \frac{7}{3}(1-0) = \boxed{\frac{7}{3}}$$