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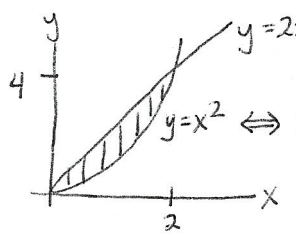
MAC 2313.9256
Cyr

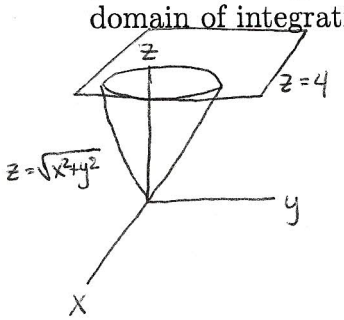
Quiz 9
You must show all work to receive full credit!!

Problem 1. (2 pts) Rewrite the integral by changing the order of integration: $\int_0^2 \int_{x^2}^{2x} f(x,y)\,dydx$. (You may find it helpful to sketch the domain of integration.)



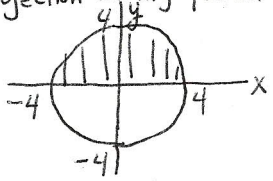
$\Rightarrow \boxed{\int_0^4 \int_{y/2}^{\sqrt{y}} f(x,y)\,dxdy}$

Problem 2. (3 pts) Set up the bounds of the triple integral $\iiint_{\mathcal{W}} y dV$ where $\mathcal{W}$ is the region bounded by the cone $z = \sqrt{x^2+y^2}$ and the plane $z = 4$ with non-negative y-coordinates ($y \geq 0$). DO NOT EVALUATE. (You may find it helpful to sketch the domain of integration.)



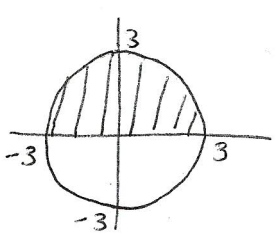
Intersection: $\sqrt{x^2+y^2} = 4$
$\Rightarrow x^2+y^2 = 16 \Rightarrow y^2 = 16-x^2 \Rightarrow y = \sqrt{16-x^2}$

Projection in x,y-plane:



$\boxed{\int_{-4}^{4} \int_{0}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{4} y\,dz\,dy\,dx}$

Problem 3. (5 pts) Evaluate by using polar coordinates: $\iint_{\mathcal{D}} (x-y)\,dA$, where $\mathcal{D} = \{x^2+y^2 \leq 9, y \geq 0\}$. (You may find it helpful to sketch the domain of integration.)



$0 \leq r \leq 3$
$0 \leq \theta \leq \pi$

$$\int_0^{\pi}\int_0^3 (r\cos\theta - r\sin\theta)\,r\,dr\,d\theta = \int_0^{\pi}\int_0^3 r^2(\cos\theta-\sin\theta)\,dr\,d\theta$$
$$= \int_0^{\pi} \frac{r^3}{3}\Big|_0^3 (\cos\theta-\sin\theta)\,d\theta = 9\int_0^{\pi}(\cos\theta-\sin\theta)\,d\theta$$
$$= 9(\sin\theta+\cos\theta)\Big|_0^{\pi} = 9(-1-1) = 9(-2) = \boxed{-18}$$