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MAC 2313.6717
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Quiz 9
You must show all work to receive full credit!!

Problem 1. (5 points) Use Lagrange multipliers to find the maximum and minimum value of $f(x,y) = xy$ subject to the constraint $4x^2 + y^2 = 8$.

$\nabla f = \langle y, x \rangle \qquad \nabla g = \langle 8x, 2y \rangle$

$y = 8\lambda x$ Note that if $x = 0$, then $y = 0$, but $(0,0)$ does not lie on constraint.

$x = 2\lambda y$ Now $\lambda = \frac{y}{8x} = \frac{x}{2y} \Rightarrow 2y^2 = 8x^2 \Rightarrow y^2 = 4x^2 \Rightarrow y = \pm 2x.$

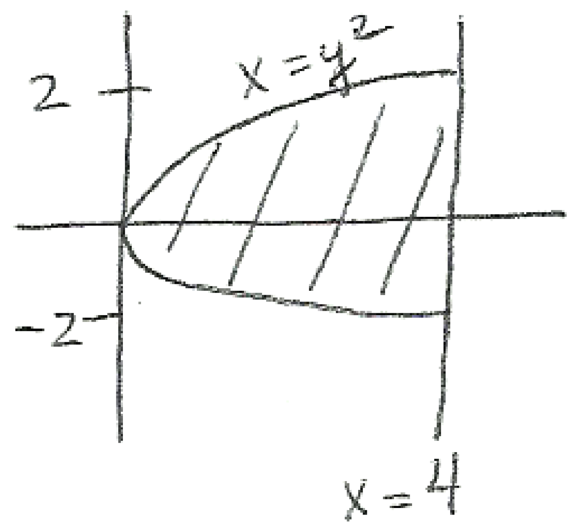
In constraint, $4x^2 + 4x^2 = 8x^2 = 8 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1.$

Critical points are $(1,2), (1,-2), (-1,-2), (-1,2)$.

$f(1,2) = f(-1,-2) = 2$ max

$f(1,-2) = f(-1,2) = -2$ min

Problem 2. (5 points) Find the volume of the solid under the surface $z = 1 + y^2$ and above the region enclosed by $x = y^2$ and $x = 4$.



$$V = \int_{-2}^{2} \int_{y^2}^{4} (1+y^2)\, dx\, dy$$

$$= \int_{-2}^{2} (1+y^2)\, x \Big|_{x=y^2}^{4} dy$$

$$= \int_{-2}^{2} (1+y^2)(4-y^2)\, dy = \int_{-2}^{2} (4 + 3y^2 - y^4)\, dy$$

$$= 4y + y^3 - \frac{1}{5}y^5 \Big|_{-2}^{2} = \left(8 + 8 - \frac{32}{5}\right) - \left(-8 - 8 + \frac{32}{5}\right)$$

$$= 32 - \frac{2}{5} \cdot 32 = \frac{3}{5} \cdot 32 = \boxed{\frac{96}{5}}$$