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MAC 2313.9728
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Quiz 9
You must show all work to receive full credit!!

Problem 1. (5 points) Use Lagrange multipliers to find the maximum and minimum value of $f(x, y, z) = 2x + 2y + z$ subject to the constraint $x^2 + y^2 + z^2 = 9$.

$\nabla f = \langle 2, 2, 1 \rangle$ $\qquad$ $\nabla g = \langle 2x, 2y, 2z \rangle$

So $2 = 2\lambda x \Rightarrow x = \frac{1}{\lambda}$ (okay since $\lambda \neq 0$). Subbing into constraint gives

$2 = 2\lambda y \Rightarrow y = \frac{1}{\lambda}$ $\qquad$ $\left(\frac{1}{\lambda}\right)^2 + \left(\frac{1}{\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 = 9 \Rightarrow$

$1 = 2\lambda z \Rightarrow z = \frac{1}{2\lambda}$ $\qquad$ $\frac{4}{4\lambda^2} + \frac{4}{4\lambda^2} + \frac{1}{4\lambda^2} = \frac{9}{4\lambda^2} = 9 \Rightarrow 4\lambda^2 = 1$

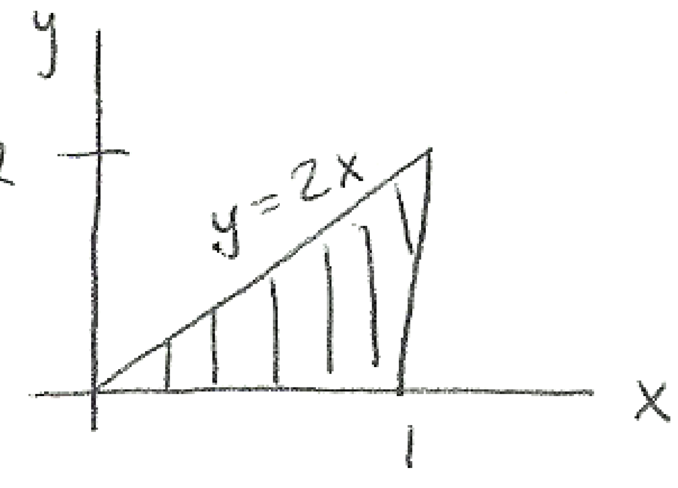
$\Rightarrow \lambda^2 = \frac{1}{4} \Rightarrow \lambda = \pm\frac{1}{2}.$

For $\lambda = \frac{1}{2}$, we have $(2, 2, 1)$ and $f(2,2,1) = 4 + 4 + 1 = 9$ max

For $\lambda = -\frac{1}{2}$, we have $(-2, -2, -1)$ and $f(-2, -2, -1) = -4 - 4 - 1 = -9$ min

Problem 2. (5 points) Evaluate by reversing the order of integration:

$$\int_0^2 \int_{y/2}^1 y \cos(x^3 - 1)\,dx\,dy$$



$= \int_0^1 \int_0^{2x} y \cos(x^3 - 1)\,dy\,dx$

$= \int_0^1 \frac{1}{2} y^2 \cos(x^3 - 1) \Big|_{y=0}^{2x} dx = \int_0^1 2x^2 \cos(x^3 - 1)\,dx$ $\qquad$ $u = x^3 - 1$, $du = 3x^2 dx$

$= \frac{2}{3} \int \cos u \, du = \frac{2}{3} \sin u = \frac{2}{3} \sin(x^3 - 1) \Big|_0^1$

$= \frac{2}{3}(\sin(0) - \sin(-1)) = \boxed{\frac{2}{3} \sin(1)}$