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MAC 2313.9722
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Quiz 9
You must show all work to receive full credit!!

Problem 1. (5 points) Use Lagrange multipliers to find the maximum and minimum value of $f(x, y, z) = xyz$ subject to the constraint $x^2 + y^2 + z^2 = 3$.

$\nabla f = \langle yz, xz, xy \rangle \qquad \nabla g = \langle 2x, 2y, 2z \rangle$

If $x, y,$ or $z = 0$, function value is 0. So assume $x, y, z \neq 0$.

$yz = 2\lambda x \Rightarrow \lambda = \frac{yz}{2x} = \frac{xz}{2y} = \frac{xy}{2z}$.

$xz = 2\lambda y$ This implies $2x^2 = 2y^2 = 2z^2 \Rightarrow x^2 = y^2 = z^2$.

$xy = 2\lambda z$

In constraint, $x^2 + x^2 + x^2 = 3x^2 = 3 \Rightarrow x^2 = 1$, so $y^2 = 1$, $z^2 = 1$ also.

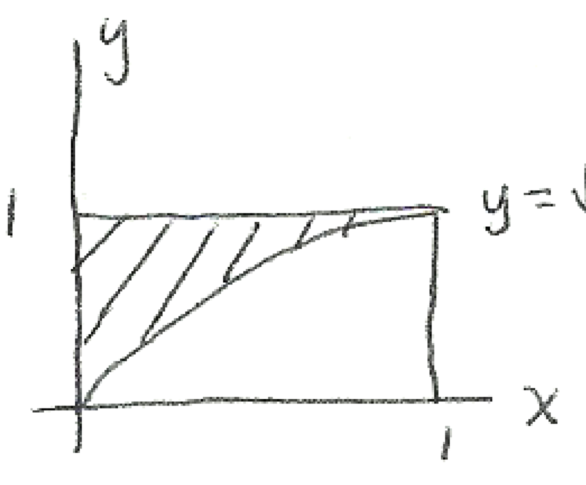
Critical points: $(\pm 1, \pm 1, \pm 1)$.

$f(1,1,1) = 1$ max

$f(-1,-1,-1) = -1$ min

Problem 2. (5 points) Evaluate by reversing the order of integration:

$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3 + 1}\,dy\,dx$$



$y = \sqrt{x} \Leftrightarrow x = y^2$

$= \int_0^1 \int_0^{y^2} \sqrt{y^3+1}\,dx\,dy = \int_0^1 y^2\sqrt{y^3+1}\,dy$.

$u = y^3 + 1 \quad du = 3y^2\,dy \qquad = \frac{1}{3}\int u^{1/2}\,du$

$= \frac{2}{9}(y^3+1)^{3/2}\Big|_0^1 = \boxed{\frac{2}{9}\left(2^{3/2} - 1\right)}$