1) Find the x-intercepts of the function \( f(x) = 3x^2 - 5x + 2 \)

Set \( f(x) = 0 \)

\[
0 = 3x^2 - 5x + 2
\]

\[
x = \frac{5 \pm \sqrt{25 - 4(3)(2)}}{6}
\]

\[
= \frac{5 \pm \sqrt{25 - 24}}{6}
\]

\[
= \frac{5 \pm 1}{6} = \frac{4}{6}, \frac{6}{6} = \frac{2}{3}, 1
\]

x-int. = \( \frac{2}{3}, \frac{5}{3}, 1 \) \((\frac{2}{3}, 0), (1, 0)\)

2) Graph the following functions. First find the vertex by completing the square or by using a formula.

a) \( f(x) = 2x^2 - 2x + 3 \)

\[
= 2(x^2 - x) + 3
\]

\[
-\frac{1}{2} + -\frac{1}{2} = -1
\]

\[
(x - \frac{1}{2})^2 = \frac{1}{4}x^2 - x \frac{1}{4}
\]

\[
= 2\left(\frac{1}{4}x^2 - x + \frac{1}{4}\right) + 3
\]

\[
= 2\left(x^2 - \frac{1}{2}\right)^2 + \frac{1}{4} + 3
\]

\[
= 2\left(x - \frac{1}{2}\right)^2 + \frac{1}{4} + 3
\]

\[f(x) = 2\left(x - \frac{1}{2}\right)^2 + \frac{7}{2}\]

vertex: \( (\frac{1}{2}, \frac{7}{2}) \)

opens up

stretched by a factor of 2
b) \( f(x) = 4x^2 + 5x - 2 \)

\[
(4x^2 + 5x + \_ - 2)
\]

So it is easier to use \( \frac{-b}{2a} \) to find the vertex, since you cannot complete the square, since you cannot find an \( a^2 \).

So \( 2a + 2a = 5 \)

\( 4a = 5 \)

No integer value that makes this true.

\[-\frac{5}{8} \]

is the \( x \)-value of the vertex.

\[
f\left(-\frac{5}{8}\right) = 4\left(-\frac{5}{8}\right)^2 + 5\left(-\frac{5}{8}\right) - 2
\]

\[
= \left(\frac{25}{16}\right) + \left(-\frac{25}{8}\right) - 2
\]

\[
= \frac{25}{16} - \frac{50}{16} - \frac{32}{16}
\]

\[
= -\frac{25 - 32}{16}
\]

\[
= -\frac{-57}{16}
\]

\[( -\frac{5}{8}, -\frac{-57}{16} ) = \text{vertex} \]
2) \( c) \quad x^2 + 2x + 1 = F(x) \)

\[ y = (x+1)^2 \quad \text{vertex: } (-1, 0) \]

no normal width, not flipped just shifted left by 1

\[ a \]

\[ \frac{-b}{2a} = \frac{5}{-2} \]

\[ f\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^2 - 5\left(-\frac{5}{2}\right) - 7 \]

\[ = \frac{-25}{4} + \frac{25}{2} \frac{1}{2} = 7 \frac{1}{4} \]

\[ \text{vertex: } \left(-\frac{5}{2}, -\frac{1}{4}\right) \]

\[ = -\frac{3}{4} \]
3) \((y, 0)\) Find the quadratic function that has \(x\)-intercepts \((1,0)\), \((5,0)\) and \(y\)-intercept \((0,10)\).

\[
(1, 0) \quad (5, 0) \quad (0, 10)
\]

\[
\frac{1}{2}
\]

\[
(x - 1)(x - 5) = \frac{y}{10}
\]

\[
y = (x - 1)(x - 5) \quad \text{when}\n\]

\[
y = x^2 - 6x + 5
\]

when \(x = 0\), \(y = 5\)

so we need to shift the equation up by 5

\[
y = x^2 - 6x + 5 \quad \text{this is only shifted up by 5}
\]
Since we want to have $a \cdot x^2 + b \cdot x + 10$ we can just multiply everything by 2

$y = 2(x^2 - 6x + 5)$

$y = 2x^2 - 12x + 10$

4) The demand function for a certain commodity is given by $p(x) = -2x + 70$, where $x$ is the # of units sold. Find the number of units that should be sold in order to maximize the revenue.

Revenue: $p(x) \cdot x$

$R(x) = x(-2x + 70)$

$R(x) = -2x^2 + 70x$

$\frac{-b}{2a} = -\frac{70}{-4} = \frac{35}{2}$

$x$-value

we have a parabola, open down. The vertex is the maximum of the parabola and thus the maximum revenue.
Since we're looking for the # of items, 4
this is our x-value.

17.5 items

but we cannot have a partial item
so we could sell either 17 or 18

\[ R(17) = -2(17)^2 + 70(17) \]
\[ = 612 \]

or

\[ R(18) = -2(18)^2 + 70(18) \]
\[ = 612 \]

both yield the same value, so we should pick the lower # since that would be fewer items to manufacture.

5) trajectory of book

maximum \[ h(t) = -8t^2 + 16t + 120 \]

(a) Find the time t, when the vertex of the book reaches the maximum height.

\[ \frac{-16}{2(-8)} = 1 \]

\[ t = 1 \]
(b) Find the maximum height.

\[ h(1) = -8(1)^2 + 16(1) + 120 \]
\[ -8 + 16 + 120 \]
\[ +8 + 120 \]

\[ h(1) = 128 \]

Maximum height: 128 units