SETS AND LOGIC (MHF 3202) – PRACTICE MIDTERM 1

EQUIVALENCES

(1) (De Morgan’s laws)
   \( \neg(P \land Q) \) is equivalent to \( \neg P \lor \neg Q \).
   \( \neg(P \lor Q) \) is equivalent to \( \neg P \land \neg Q \).

(2) (Commutative laws)
   \( P \land Q \) is equivalent to \( Q \land P \).
   \( P \lor Q \) is equivalent to \( Q \lor P \).

(3) (Associative laws)
   \( P \land (Q \land R) \) is equivalent to \( (P \land Q) \land R \).
   \( P \lor (Q \lor R) \) is equivalent to \( (P \lor Q) \lor R \).

(4) (Idempotent laws)
   \( P \land P \) is equivalent to \( P \).
   \( P \lor P \) is equivalent to \( P \).

(5) (Distributive laws)
   \( P \land (Q \lor R) \) is equivalent to \( (P \land Q) \lor (P \land R) \).
   \( P \lor (Q \land R) \) is equivalent to \( (P \lor Q) \land (P \lor R) \).

(6) (Absorption laws)
   \( P \lor (P \land Q) \) is equivalent to \( P \).
   \( P \land (P \lor Q) \) is equivalent to \( P \).

(7) (Double Negation law)
   \( \neg \neg P \) is equivalent to \( P \).

(8) (Tautology laws)
   \( P \land (\text{a tautology}) \) is equivalent to \( P \).
   \( P \lor (\text{a tautology}) \) is a tautology.
   \( \neg(\text{a tautology}) \) is a contradiction.

(9) (Contradiction laws)
   \( P \land (\text{a contradiction}) \) is a contradiction.
   \( P \lor (\text{a contradiction}) \) is equivalent to \( P \).
   \( \neg(\text{a contradiction}) \) is a tautology.

(10) (Conditional law)
     \( P \rightarrow Q \) is equivalent to \( \neg P \lor Q \).
Question 1. Circle the correct answer.

(1) (1 point)
Is the following valid reasoning?
“If I answer this question correctly, I will get a point. Therefore I will get a point.”
True.
False.

(2) (1 point)
Is the formula \((P \lor Q) \land \neg Q\) equivalent to \(P \land \neg Q\)?
True.
False.

(3) (1 point)
Is the emptyset the truth set for the statement \(P(x)\) saying "\(x\) is an even prime number." in the universe of natural numbers?
True.
False.

(4) (1 point)
Let \(A, B, C\) be sets. Is the following equality true?
\[
A \setminus B \cup B \setminus C = A \setminus C
\]
True.
False.

(5) (1 point)
Is there more than one free variable in the following statement?
\[
w \in \{ x : x = 2k + 1 \}
\]
True.
False.

Question 2. (10 points) Use a truth table to verify the Absorption law \(P \lor (P \land Q)\) is equivalent to \(P\).
Question 3. (15 point) Let $A, B, C$ be sets. Use logical equivalences to show that 
$$(A \setminus B) \cap C = (A \cap C) \setminus (B \cap C).$$

Question 4. (15 points) Make a truth table for the formula 
$$( (P \rightarrow Q) \land (Q \rightarrow R) ) \rightarrow (P \rightarrow R).$$

Decide whether the formula above is a tautology, contradiction, or neither.

Question 5.

(a) (2 points) Analyze the logical form of the following statement: Both having a fever and having a headache are sufficient conditions for George to go to the doctor.

(b) (3 points) Translate the following statement into idiomatic English.

$$\exists x \forall y P(x, y),$$

where $P(x, y)$ means "$x$ is a parent of $y$".