

**ADVANCED CALCULUS MAA4102**  
**FIRST HOUR EXAM**  
**FALL 2005**

Name:

No calculators permitted during the exam.

Each problem is worth 20 points.

Explain all answers!

1.

a. Give a careful definition of what it means for a sequence to converge to a number  $L$ .

b. Using the DEFINITION of limit show that  $\lim_{n \rightarrow \infty} \frac{5n+3}{2n+7} = \frac{5}{2}$ .

c. Using limit theorems compute  $\lim_{n \rightarrow \infty} (\sqrt{n^2 + 3n} - n)$ .

2.

a. State the square root algorithm of Archimedes/Heron.

b. Use the square root algorithm of Archimedes/Heron to compute three approximations of  $\sqrt{7}$ .

c. If  $K > 0$  and  $x_n$  is the  $n^{\text{th}}$  term in the Archimedes/Heron algorithm to approximate  $\sqrt{K}$ , then show that  $x_{n+1} \geq \sqrt{K}$ .

3.

a. Give a careful statement of the least upper bound principle.

b. Prove: If a sequence is bounded and increasing, then it converges.

4.

a. Give a careful definition of what it means for a sequence to be *Cauchy*.

b. Prove: If a sequence is Cauchy, then it is bounded.

5.

a. Prove: If a sequence is bounded, then it has a convergent subsequence.

b. Prove: If a sequence is Cauchy, then it has a convergent subsequence.